Principles of Engineering Mechanics

Topic Objective:

At the end of this topic student would be able to:

- Define the term mechanics
- Describe the engineering and mechanics relationship
- Highlight the development of the field of mechanics

Definition/Overview:

Mechanics: Mechanics is the branch of physics concerned with the behaviour of physical bodies when subjected to forces or displacements, and the subsequent effect of the bodies on their environment. The discipline has its roots in several ancient civilizations (see History of classical mechanics and Timeline of classical mechanics). During the early modern period, scientists such as Galileo, Kepler, and especially Newton, laid the foundation for what is now known as classical mechanics.

Key Points:

1. Engineering and Mechanics

Mechanical Engineering is an engineering discipline that involves the application of principles of physics for analysis, design, manufacturing, and maintenance of mechanical systems. Mechanical engineering is one of the oldest and broadest engineering disciplines.

It requires a solid understanding of core concepts including mechanics, kinematics, thermodynamics, fluid mechanics, and energy. Mechanical engineers use the core principles as well as other knowledge in the field to design and analyze motor vehicles, aircraft, heating and cooling systems, watercraft, manufacturing plants, industrial equipment and machinery, robotics, medical devices and more.

2. Development

Applications of mechanical engineering are found in the records of many ancient and medieval societies throughout the globe. In ancient Greece, the works of Archimedes (287 BC212 BC) and Heron of Alexandria (c. 1070 AD) deeply influenced mechanics in the Western tradition. In China, Zhang Heng (78139 AD) improved a water clock and invented a seismometer, and Ma Jun (200265 AD) invented a chariot with differential gears. The medieval Chinese horologist and engineer Su Song (10201101 AD) incorporated an escapement mechanism into his astronomical clock tower two centuries before any escapement could be found in clocks of medieval Europe, as well as the world's first known endless power-transmitting chain drive.

During the years from 7th to 15th century, the era called the Islamic golden age, there have been remarkable contributions from Muslims in the field of mechanical technology, Al Jaziri, who was one of them wrote his famous "Book of Knowledge of Ingenious Mechanical Devices" in 1206 presented many mechanical designs. He is also considered to be the inventor of such mechanical devices which now form the very basic of mechanisms, such as crank and cam shafts.

During the early 19th century in England and Scotland, the development of machine tools led mechanical engineering to develop as a separate field within engineering, providing manufacturing machines and the engines to power them. The first British professional society of mechanical engineers was formed in 1847, thirty years after civil engineers formed the first such professional society. In the United States, the American Society of Mechanical Engineers (ASME) was formed in 1880, becoming the third such professional engineering society, after the American Society of Civil Engineers (1852) and the American Institute of Mining Engineers (1871). The first schools in the United States to offer an engineering education were the United States Military Academy in 1817, an institution now known as Norwich University in 1819, and Rensselaer Polytechnic Institute in 1825. Education in mechanical engineering has historically been based on a strong foundation in mathematics and science. The field of mechanical engineering is considered among the broadest of engineering disciplines. The work of mechanical engineering ranges from the ocean bottoms to space.

Mechanics is, in the most general sense, the study of forces and their effect upon matter. Typically, engineering mechanics is used to analyze and predict the acceleration and deformation (both elastic and plastic) of objects under known forces (also called loads) or stresses. Subdisciplines of mechanics include:

- Statics, the study of non-moving bodies under known loads
- Dynamics (or kinetics), the study of how forces affect moving bodies
- Mechanics of materials, the study of how different materials deform under various types of stress
- Fluid mechanics, the study of how fluids react to forces
- Continuum mechanics, a method of applying mechanics that assumes that objects are continuous (rather than discrete)

Mechanical engineers typically use mechanics in the design or analysis phases of engineering. If the engineering project were the design of a vehicle, statics might be employed to design the frame of the vehicle, in order to evaluate where the stresses will be most intense. Dynamics might be used when designing the car's engine, to evaluate the forces in the pistons and cams as the engine cycles. Mechanics of materials might be used to choose appropriate materials for the frame and engine. Fluid mechanics might be used to design a ventilation system for the vehicle, or to design the intake system for the engine.

3. Kinematics

Kinematics is the study of the motion of bodies (objects) and systems (groups of objects), while ignoring the forces that cause the motion. The movement of a crane and the oscillations of a piston in an engine are both simple kinematic systems. The crane is a type of open kinematic chain, while the piston is part of a closed four bar linkage.

Mechanical engineers typically use kinematics in the design and analysis of mechanisms. Kinematics can be used to find the possible range of motion for a given mechanism, or, working in reverse, can be used to design a mechanism that has a desired range of motion.

4. Gravitation

Gravitation is a natural phenomenon that gives weight to objects. In everyday life, attraction due to gravity is the result of the presence of relatively large bodies, such as the Earth and the Moon. Gravitation not only causes attraction to very large bodies, but can also distort the surface of planets and other natural satellites, causing tides, earthquakes, and in extreme cases even volcanic eruptions as found on Jupiter's closest-orbiting moon, Io. In outer space, gravity between particles of interstellar dust gives rise to stars and planets. The very existence of the Sun and every star in the universe is the consequence of inelastic collisions caused by internal friction of dusty nebulas. As this occurs, flows known as convection (by which fluid flow occurs under the influence of a temperature gradient and gravity) distribute the heat caused by friction.

Gravity not only causes planets and stars to move in predictable orbital paths, but it is also the only force known to be capable of forming planets, stars, and galaxies. The gravitational pressure inside the centers of stars can merge pairs of atoms to produce the variety of elements on the periodic table, and in doing so can generate temperatures of millions of degrees inside the center of a star, in contrast to the thousands of degrees generated inside the core of planets. By stimulating the interiors of some large very stars, the force of gravity can trigger supernovas - very powerful explosions capable of destroying nearby planets or even other solar systems with extremely fast and hot solar winds and cosmic rays. Many galaxies parallel this phenomenon at much larger scales of both distance and time, particularly in active galaxies, such as radio galaxies.

The terms gravitation and gravity are mostly interchangeable in everyday use, but a distinction may be made in scientific usage. "Gravitation" is a general term for the attraction that bodies with mass have to one another, while "gravity" refers specifically to the net force bodies such as the Earth have on objects in their vicinity, including the effect of the Earth's rotation. Modern physics describes gravitation using the general theory of relativity, in which gravitation is a consequence of the curvature of spacetime, which governs the motion of inertial objects. The simpler Newton's law of universal gravitation provides an excellent approximation for most calculations.

5. Newtonian Gravitation

In 1687, English mathematician Sir Isaac Newton published Principia, which hypothesizes the inverse-square law of universal gravitation. In his own words, I deduced that the forces which keep the planets in their orbs must be reciprocally as the squares of their distances from the centers about which they revolve; and thereby compared the force requisite to keep the Moon in her orb with the force of gravity at the surface of the Earth; and found them answer pretty nearly. Forty-two years earlier Ismal Bullialdus had proposed much the same theory. Newton's theory enjoyed its greatest success when it was used to predict the existence of Neptune based on motions of Uranus that could not be accounted by the actions of the other planets. Calculations by John Couch Adams and Urbain Le Verrier both predicted the general position of the planet, and Le Verrier's calculations are what led Johann Gottfried Galle to the discovery of Neptune.

Ironically, it was another discrepancy in a planet's orbit that helped to point out flaws in Newton's theory. By the end of the 19th century, it was known that the orbit of Mercury showed slight perturbations that could not be accounted for entirely under Newton's theory, but all searches for another perturbing body (such as a planet orbiting the Sun even closer than Mercury) had been fruitless. The issue was resolved in 1915 by Albert Einstein's new General Theory of Relativity, which accounted for the small discrepancy in Mercury's orbit. Although Newton's theory has been superseded, most modern non-relativistic gravitational calculations are still made using Newton's theory because it is a much simpler theory to work with than General Relativity, and gives sufficiently accurate results for most applications.

6. General relativity

In general relativity, the effects of gravitation are ascribed to spacetime curvature instead of a force. The starting point for general relativity is the equivalence principle, which equates free fall with inertial motion, and describes free-falling inertial objects as being accelerated relative to non-inertial observers on the ground. In Newtonian physics, however, no such acceleration can occur unless at least one of the objects is being operated on by a force. Einstein proposed that spacetime is curved by matter, and that free-falling objects are moving along locally straight paths in curved spacetime. These straight lines are called geodesics. Like Newton's First Law, Einstein's theory stated that if there is a force applied to an object, it would deviate from the geodesics in spacetime. For example, we are no longer following the geodesics while standing because the mechanical resistance of the Earth exerts an upward force on us. Thus, we are non-inertial on the ground. This explains why moving along the geodesics in spacetime is considered inertial. Einstein discovered the field equations of general relativity, which relate the presence of matter and the curvature of spacetime and are named after him. The Einstein field equations are a set of 10 simultaneous, non-linear, differential equations. The solutions of the field equations are the components of the metric tensor of spacetime. A metric tensor describes a geometry of spacetime. The geodesic paths for a spacetime are calculated from the metric tensor.

Notable solutions of the Einstein field equations include:

- The Schwarzschild solution, which describes spacetime surrounding a spherically symmetric non-rotating uncharged massive object. For compact enough objects, this solution generated a black hole with a central singularity. For radial distances from the center which are much greater than the Schwarzschild radius, the accelerations predicted by the Schwarzschild solution are practically identical to those predicted by Newton's theory of gravity.
- The Reissner-Nordstrm solution, in which the central object has an electrical charge. For charges with a geometrized length which are less than the geometrized length of the mass of the object, this solution produces black holes with two event horizons.
- The Kerr solution for rotating massive objects. This solution also produces black holes with multiple event horizons.

- The Kerr-Newman solution for charged, rotating massive objects. This solution also produces black holes with multiple event horizons.
- The cosmological Robertson-Walker solution, which predicts the expansion of the universe.

The tests of general relativity included:

- General relativity accounts for the anomalous perihelion precession of Mercury.
- The prediction that time runs slower at lower potentials has been confirmed by the Pound-Rebka experiment, the Hafele-Keating experiment, and the GPS.
- The prediction of the deflection of light was first confirmed by Arthur Eddington in 1919.The Newtonian corpuscular theory also predicted a lesser deflection of light, but Eddington found that the results of the expedition confirmed the predictions of general relativity over those of the Newtonian theory. However this interpretation of the results was later disputed. More recent tests using radio interferometric measurements of quasars passing behind the Sun have more accurately and consistently confirmed the deflection of light to the degree predicted by general relativity.
- The time delay of light passing close to a massive object was first identified by Irwin Shapiro in 1964 in interplanetary spacecraft signals.
- Gravitational radiation has been indirectly confirmed through studies of binary pulsars.
- Alexander Friedmann in 1922 found that Einstein equations have non-stationary solutions (even in the presence of the cosmological constant). In 1927 Georges Lematre showed that static solutions of the Einstein equations, which are possible in the presence of the cosmological constant, are unstable, and therefore the static universe envisioned by Einstein could not exist. Later, in 1931, Einstein himself agreed with the results of Friedmann and Lematre. Thus general relativity predicted that the Universe had to be non-staticit had to either expand or contract. The expansion of the universe discovered by Edwin Hubble in 1929 confirmed this prediction

7. Gravitational radiation

In general relativity, gravitational radiation is generated in situations where the curvature of spacetime is oscillating, such as is the case with co-orbiting objects. The gravitational radiation emitted by the solar system is far too small to measure. However, gravitational radiation has been indirectly observed as an energy loss over time in binary pulsar systems such as PSR 1913+16. It is believed that neutron star mergers and black hole formation may create detectable amounts of gravitational radiation. Gravitational radiation observatories such as LIGO have been created to study the problem. No confirmed detections have been made of this hypothetical radiation, but as the science behind LIGO is refined and as the instruments themselves are endowed with greater sensitivity over the next decade, this may change.

In Section 2 of this course you will cover these topics:

- Vectors
- Forces

Topic Objective:

At the end of this topic student would be able to:

- Define the term vectors
- Describe the magnitude of the vector
- Highlight the representation of a vector

Definition/Overview:

Vectors: In elementary mathematics, physics, and engineering, a vector (sometimes called a geometric or spatial vector) is a geometric object that has both a magnitude (or length), direction and sense, i.e., orientation along the given direction. A vector is frequently represented by a line segment with a definite direction, or graphically as an arrow, connecting an initial point A with a terminal point B, and denoted by

Key Points:

1. Vectors

The magnitude of the vector is the length of the segment and the direction characterizes the displacement of B relative to A: how much one should move the point A to "carry" it to the point B. Many algebraic operations on real numbers have close analogues for vectors. Vectors can be added, subtracted, multiplied by a number, and flipped around so that the direction is reversed. These operations obey the familiar algebraic laws: commutativity, associativity, distributivity. The sum of two vectors with the same initial point can be found geometrically using the parallelogram law. Multiplication by a positive number, commonly called a scalar in this context, amounts to changing the magnitude of vector, that is, stretching or compressing it while keeping its direction; multiplication by negative numbers changes the magnitude and reverses the direction.

Cartesian coordinates provide a systematic way of describing vectors and operations on them. A vector becomes a tuple of real numbers, its scalar components. Addition of vectors and multiplication of a vector by a scalar are simply done component by component, see coordinate vector. Vectors play an important role in physics: velocity and acceleration of a moving object and forces acting on a body are all described by vectors. Many other physical quantities can be usefully thought of as vectors. The mathematical representation of a physical vector depends on the coordinate system used to describe it. Other vector-like objects that describe physical quantities and transform in a similar way under changes of the coordinate system include pseudovectors and tensors.

In this context, a vector is a geometric entity characterized by a magnitude (in mathematics a number, in physics a number times a unit) and a direction, often represented graphically by an arrow. When it becomes necessary to distinguish it from vectors as defined elsewhere, this is sometimes referred to as a **geometric**, **spatial**, or **Euclidean** vector. When a vector is thought of as an arrow in Euclidean space, it possesses a definite **initial point** and **terminal point**. Such a vector is called a **bound vector**. In other situations, when only the magnitude and direction of the vector matter, then the particular initial point is of no importance, and the vector is called a **free vector**. Thus two arrows and in space represent the same free vector if they have the same magnitude and direction: equivalently, they are equivalent if the quadrilateral *ABB'A'* is a parallelogram. If the Euclidean space is equipped with a choice of origin, then a free vector is equivalent to the bound vector of the same magnitude and direction whose initial point is the origin. The term *vector* also has generalizations to larger dimensions and to more formal approaches with much wider applications.

2. Representation of a vector

Vectors are usually denoted in lowercase boldface, as **a** or lowercase italic boldface, as ***a***. (Uppercase letters are typically used to represent matrices.) Other conventions include or <u>*a*</u>, especially in handwriting. Alternately, some use a tilde (~) or a wavy underline drawn beneath the symbol, which is a convention for indicating boldface type. If the vector represents a directed distance or displacement from a point *A* to a point *B* (see figure), it can also be denoted as or <u>*AB*</u>. The hat symbol (^) is typically used to denote unit vectors

(vectors with unit length), as in . Vectors are usually shown in graphs or other diagrams as arrows (directed line segments), as illustrated in the figure. Here the point *A* is called the *origin*, *tail*, *base*, or *initial point*; point *B* is called the *head*, *tip*, *endpoint*, *terminal point* or *final point*. The length of the arrow is proportional to the vector's magnitude, while the direction in which the arrow points indicates the vector's direction.

On a two-dimensional diagram, sometimes a vector perpendicular to the plane of the diagram is desired. These vectors are commonly shown as small circles. A circle with a dot at its centre (Unicode U+2299 ⊙) indicates a vector pointing out of the front of the diagram, toward the viewer. A circle with a cross inscribed in it (Unicode U+2297 ⊗) indicates a vector pointing into and behind the diagram. These can be thought of as viewing the tip of an arrow head on and viewing the vanes of an arrow from the back.

In order to calculate with vectors, the graphical representation may be too cumbersome. Vectors in an *n*-dimensional Euclidean space can be represented in a Cartesian coordinate system. The endpoint of a vector can be identified with an ordered list of *n* real numbers (*n*-tuple). As an example in two dimensions (see figure), the vector from the origin *O* = (0,0) to the point *A* = (2,3) is simply written as:

The notion that the tail of the vector coincides with the origin is implicit and easily understood. Thus, the more explicit notation is usually not deemed necessary and very rarely used. In three dimensional Euclidean space (or), vectors are identified with triples of numbers corresponding to the Cartesian coordinates of the endpoint (*a*,*b*,*c*):

These numbers are often arranged into a column vector or row vector, particularly when dealing with matrices, as follows:

Another way to express a vector in three dimensions is to introduce the three standard basis vectors:

These have the intuitive interpretation as vectors of unit length pointing up the *x*, *y*, and *z* axis of a Cartesian coordinate system, respectively, and they are sometimes referred to as versors of those axes. In terms of these, any vector in can be expressed in the form:

In introductory physics classes, these three special vectors are often instead denoted (or), but such notation clashes with the index notation and the summation convention commonly used in higher level mathematics, physics, and engineering. The use of Cartesian versors such as as a basis in which to represent a vector is not mandated. Vectors can also be expressed in terms of cylindrical unit vectors or spherical unit vectors . The latter two choices are more convenient for solving problems which possess cylindrical or spherical symmetry respectively.

3. Vector components

A **component** of a vector is the influence of that vector in a given direction. Components are themselves vectors. A vector is often described by a fixed number of components that sum up into this vector uniquely and totally. When used in this role, the choice of their constituting directions is dependent upon the particular coordinate system being used, such as Cartesian coordinates, spherical coordinates or polar coordinates. For example, an **axial component** of a vector is a component whose direction is determined by a projection onto one of the Cartesian coordinate axes, whereas **radial** and **tangential components** relate to the *radius of rotation* of an object as their direction of reference. The former is parallel to the radius and the latter is orthogonal to it. Both remain orthogonal to the *axis of rotation* at all times. (In two dimensions this requirement becomes redundant as the axis degenerates to a *point of rotation.*) The choice of a coordinate system doesn't affect properties of a vector or its behaviour under transformations.

4. Vectors as directional derivatives

A vector may also be defined as a **directional derivative**: consider a function $f(x^\alpha)$ and a curve $x^\alpha(\tau)$. Then the directional derivative of f is a scalar defined as where the index α is summed over the appropriate number of dimensions (for example, from 1 to 3 in 3-dimensional Euclidean space, from 0 to 3 in 4-dimensional spacetime, etc.). Then consider a vector tangent to $x^{\alpha}(\tau)$:

The directional derivative can be rewritten in differential form (without a given function *f*) as

Therefore any directional derivative can be identified with a corresponding vector, and any vector can be identified with a corresponding directional derivative. A vector can therefore be defined precisely as

5. Vectors, pseudovectors, and transformations

An alternative characterization of Euclidean vectors, especially in physics, describes them as lists of quantities which behave a certain way under a coordinate transformation. A *contravariant vector* is required to have components that "transform like the coordinates" under changes of coordinates such as rotation and dilation. The vector itself does not change under these operations; instead, the components of the vector make a change that cancels the change in the spatial axes, in the same way that co-ordinates change. In other words, if the reference axes were rotated in one direction, the component representation of the vector would rotate in exactly the opposite way. Similarly, if the reference axes were stretched in one direction, the components of the vector, like the co-ordinates, would reduce in an exactly compensating way. Mathematically, if the coordinate system undergoes a transformation described by an invertible matrix M, so that a coordinate vector $\mathbf{x}$ is transformed to $\mathbf{x}' = M\mathbf{x}$, then a contravariant vector $\mathbf{v}$ must be similarly transformed via $\mathbf{v}' = M\mathbf{v}$. This important requirement is what distinguishes a contravariant vector from any other triple of physically meaningful quantities. For example, if v consists of the x, y, and z-components of velocity, then v is a contravariant vector: When space is stretched, rotated, or twisted, then the components of the velocity transform in the same way as space. On the other hand, for instance, a triple consisting of the length, width, and height of a rectangular box could make up the three components of an abstract vector, but this vector would not be contravariant, since rotating the box does not change the box's length, width, and height! Examples of contravariant vectors include displacement, velocity, electric field, momentum, force, and acceleration.

In the language of differential geometry, the requirement that the components of a vector transform according to the same matrix of the coordinate transition is equivalent to defining a *contravariant vector* to be a tensor of contravariant rank one. Alternatively, a contravariant vector is defined to be a tangent vector, and the rules for transforming a contravariant vector follow from the chain rule. Some vectors transform like contravariant vectors, except that when they are reflected through a mirror, they flip *and* gain a minus sign. A transformation that switches right-handedness to left-handedness and vice versa like a mirror does is said to change the *orientation* of space. A vector which gains a minus sign when the orientation of space changes is called a **pseudovector** or an **axial vector**. Ordinary vectors are sometimes called **true vectors** or **polar vectors** to distinguish them from pseudovectors. Pseudovectors occur most frequently as the cross product of two ordinary vectors.

One example of a pseudovector is angular velocity. Driving in a car, and looking forward, each of the wheels has an angular velocity vector pointing to the left. If the world is reflected in a mirror which switches the left and right side of the car, the *reflection* of this angular velocity vector points to the right, but the *actual* angular velocity vector of the wheel still points to the left, corresponding to the minus sign. Other examples of pseudovectors include magnetic field, torque, or more generally any cross product of two (true) vectors.

Topic Objective:

At the end of this topic student would be able to:

- Define the term forces
- Describe the gravitation and force relationship
- Highlight the types of forces

Definition/Overview:

Forces: a force is that which can cause an object with mass to change its velocity. Force has both magnitude and direction, making it a vector quantity. Newton's second law states that an object with a constant mass will accelerate in proportion to the net force acting upon and in inverse proportion to its mass. Equivalently, the net force on an object equals the rate at which its momentum changes

Key Points:

1. Forces

One of the basic features in physics is the occurrence of forces that keep matter together. There are for example, the forces that keep the cells together to build up the human body, and there is the gravitational force that keeps us on the ground and the moon in orbit around the earth. We can ourselves exert forces when we push something and, by engineering, get some of the energy content in oil to produce a force on the wheels of a car to move it. From the macroscopic point of view we can imagine many different kinds of forces, forces that act at impact but also forces that act over a distance such as the gravitational one. In physics, though, we try to systematise and to find as many general concepts as possible. One such systematisation is to find out the ultimate constituents of matter. Another is to find out the forces that act between them. In the first case, we have been able to divide up matter into atoms and the atoms into nuclei and electrons, and then the nuclei into protons and neutrons. By colliding protons with protons or protons with electrons, particle physicists have uncovered that all matter can be built from a number of quarks (a concept introduced by Murray Gell-Mann in the 60's) and leptons (electrons and neutrinos and their heavier cousins). In the same process physicists have uncovered four basic forces that act between these matter particles - gravitation, electromagnetism, the strong and the weak nuclear force. Only the first two can be directly seen in the macroscopic world so let us first describe them.

2. Gravitation

The first quantitative theory of gravitation based on observations was formulated by Isaac Newton in 1687 in his Principia. He wrote that the gravity force that acts on the sun and the planets depends on the quantity of matter that they contain. It propagates to large distances and diminishes always as the inverse of the square of the distance. The formula for the force F between two objects with masses m_1 and m_2 a distance r away is thus

$$F=Gm_1m_2/r^2,$$

Where, *G* is a constant of proportionality, the gravitational constant. Newton was not fully happy with his theory since it assumed an interaction over a distance. This difficulty was removed when the concept of the gravity field was introduced, a field that permeates space. Newton's theory was very successfully applied to celestial mechanics during the 18th and the beginning of the 19th century. For example J.C. Adams and U.J.J. Leverrier were able to conjecture a planet outside of Uranus from irregularities in its orbit and subsequently, Neptune was found. One problem remained though. Leverrier had in 1845 calculated that Mercury's orbit precesses 35" per century in contrast to the Newtonian value that is zero. Later measurements gave a more precise value of 43". (The observed precession is really 5270"/century, but a painstaking calculation to subtract the disturbances from all the other planets gives the value of 43".) It was not until 1915 that Albert Einstein could explain this discrepancy.

Galilei was the first to observe that objects seemingly fall at the same speed regardless of their masses. In Newton's equations the concept of mass occurs in two different equations. The second law says that a force *F* on a body with mass *m* gives an acceleration *a* according to the equation $F=ma$. In the law of gravity, the force of gravity *F* satisfies $F=mg$, where *g* depends on the other bodies exerting a force on the body (the earth usually, when we talk of the gravity force). In both equations *m* is a proportionality factor (the inertial mass and the gravitational mass) and there is no obvious reason that they should be the same for two different objects. However, all experiments indicate that they are. Einstein took this fact as the starting point for his theory of gravitation. If you cannot distinguish the inertial mass from the gravitational one you cannot distinguish gravitation from an acceleration. An experiment performed in a gravity field could instead be performed in an accelerating elevator with no gravity field. When an astronaut in a rocket accelerates to get away from earth he feels a gravity force that is several times that on earth. Most of it comes from the acceleration. If one cannot distinguish gravity from acceleration one can always substitute the gravity force by being in an accelerating frame. A frame in which the acceleration cancels the gravity force is called an inertial frame. Hence the moon orbiting the earth can instead be regarded to be in an accelerating frame. However this frame will be different from point to point since the gravity field changes. (In the example with the moon the gravity field changes direction from one point to another.) The principle that one can always find an inertial frame at every point of space and time in which physics follows the laws in the absence of gravitation is called the *Equivalence Principle.*

The fact that the gravitational force can be thought of as coordinate systems that differ from point to point means that gravity is a geometric theory. The true coordinate system that covers the whole of space and time is hence a more complex one than the ordinary flat ones we are used to from ordinary geometry. This type of geometry is called *Non Euclidean Geometry.* The force as we see it comes from properties of space and time. We say that space-time is curved. Consider a ball lying on a flat surface. It will not move, or if there is no friction, it could be in a uniform movement when no force is acting on it. If the surface is curved, the ball will accelerate and move down to the lowest point choosing the shortest path. Similarly, Einstein taught us that the four-dimensional space and time is curved and a body moving in this curved space moves along a *geodesics* which is the shortest path. Einstein showed that the gravity field is the geometric quantity that defines the so-called proper time, which is a concept that takes the same value in all coordinate systems similar to distance in ordinary space. He also managed to construct equations for the gravity field, the celebrated *Einstein's equations,* and with these equations he could compute the correct value for the precession for the orbit of Mercury. The equations also give the measured value of the deflection of light rays that pass the sun and there is no doubt that the equations give the correct results for macroscopic gravitation. Einstein's theory of gravitation, or *General Relativity,* as he called it himself is one of the greatest triumphs of modern science.

3. Electromagnetism

It was James Clark Maxwell who, in 1865, finally unified the concepts of electricity and magnetism into one theory of electromagnetism. The force is mediated by the electromagnetic field. The various derivatives of this field lead to the electric and the magnetic fields, respectively. The theory is not totally symmetric in the electric and the magnetic fields though, since it only introduces direct sources to the electric field, the electric charges. A fully symmetric theory would also introduce magnetic charges, (predicted to exist by modern quantum theory but with such huge magnitudes that free magnetic charges must be extremely rare in our universe). For two static bodies with charges e1 and e2 the theory leads to Coulomb's Law giving the force between the two bodies

$$F=ke_1e_2/r^2,$$

Where, again *k* is a proportionality constant. Note the resemblance with Newton's law for gravity. There is one difference though. While the gravitational force always is attractive, the electromagnetic one can also be repulsive. The charges can either have negative signs such as for the electron or be positive as for the proton. This leads to the fact that positive and negative charges tend to bind together such as in the atoms and hence, screen each other and reduce the electromagnetic field. Most of the particles in the earth screen each other in this way and the total electromagnetic field is very much reduced. Even so we know of the magnetic field of the earth. Also in our bodies most charges are screened so there is a very minute electromagnetic force between a human being and the earth. The situation is very different for the gravity field. Since it is always attractive, every particle in the earth interacts with every particle in a human body, setting up a force with is just our weight. However, if we compare the electromagnetic and the gravitational forces between two electrons we will find that the electromagnetic one is bigger by a factor which is roughly 10^{40}. This is an unbelievably large number! It shows that when we come to microcosm and study the physics of elementary particles we do not need to consider gravity when we study quantum electrodynamics, at least not at ordinary energies.

When examining Maxwell's equations one finds that the electromagnetic field travels with a finite velocity. This means that *Coulomb's Law* is only true once the electromagnetic field has had time to travel between the two charges. It is a static law. One also finds that the electromagnetic field travels as a wave just in the same way as light does. It was Rmer who discovered that the velocity of light is finite and Newton and Huygens who discovered that light travels as waves in the late 17th century, and by the end of the 19th century the velocity of light was well established and seen to agree with the velocity of the electromagnetic field. Hence it was established that light is nothing but electromagnetic radiation. In 1900 Max Planck proposed that light is quantised in order to explain the black body radiation. However, it was Albert Einstein who was the first to really understand the revolutionary consequences of this idea when he formulated the *photoelectric* effect. The electromagnetic field can be understood as a stream of corpuscular bodies to be called *photons* that make up the electromagnetic field. The revolutionary aspect of this idea was that a stream of particles also could behave as a wave and there was much opposition to the idea from many established scientists of the day. It was not until 1923 when Arthur Compton experimentally showed that a light quanta could deflect an electron just like a corpuscular body would do it, that this debate was over.

If we think about the electric force between two charges as the electromagnetic field mediating it over a distance, we can now get a more fundamental picture as a stream of photons sent out from one particle to hit the other. This is a more intuitive picture than a force acting over a distance. Our macroscopic picture of a force is that something hits a body that then feels a force. In the microscopic world this is then again a way to understand a force. However, it is more complex. Suppose there are two charged particles that interact. Which particle is sending out a photon and which is receiving the photon if the two particles are identical as quantum mechanics tells us about fundamental particles? The answer must be that the picture should include both possibilities. The discovery that the electromagnetic field is quantised started the development of quantum mechanics and led us to a microcosm that is just built up by point-like objects and where forces occur when two particles hit each other.

Quantum mechanics as such led to many new revolutionary concepts. One of the most important ones is *Heisenberg's Uncertainty Relation* formulated by Werner Heisenberg in 1927, which states that one cannot measure position and momentum or energy and time exactly simultaneously. For a nucleus, one can either determine the position of an electron and know nothing of its momentum or know its momentum and nothing about its position. In the picture showing the force field between two charges, we should think of it as photons travelling from one charge to another. Hence the energy cannot be determined better than what the uncertainty relation tells us because of the uncertainty in the determination of the time. Hence the special relativity relation for light that the photon is massless which translates into the relation that the energy^2=momentum^{2}c^2 need not be satisfied. If we put the energy and the tree-dimensional momentum together into the four-momentum we see that it is not constrained by the masslessness condition, we say that the photon is virtual and consequently has a (virtual) mass. We can thus interpret the process above as either a certain photon going from particle 1 to particle 2 with a certain four-momentum or as one from particle 2 to particle 1 with the opposite four-momentum. When two charges are far away the uncertainty relation gives little freedom and the photon is closer to masslessness, We know that *Coulomb's law* seems to be valid at the longest distances so it must be set up by the photons close to masslessness. If two charges are close there should be more terms to the force. Incidentally in order to measure the velocity of light the photons must interact. Hence there is a slight uncertainty in its mass and a slight uncertainty in its velocity. However, we measure always the same velocity for light which means that at the macroscopic distances that we measure, the virtuality and hence the mass of the photon is essentially zero to a very good accuracy. It is then consistent to say that the velocity of light is constant.

The full description of the electromagnetic force between elementary particles was formulated by Sin-Itiro Tomonaga, Richard Feynman and Julian Schwinger in independent works in the 1940's. They formulated *Quantum ElectroDynamics (QED).* This is a theory that takes full account of quantum physics and special relativity (which is the underlying symmetry of *Maxwell's Equations*). It is very elegantly formulated by so-called *Feynman diagrams,* where the elementary particles exchange photons as was described above and where each diagram constitutes a certain mathematical expression that can be obtained from some basic rules for the propagation of virtual particles and from the interaction vertices. The simplest diagram for the interaction between two electrons is:

This diagram in fact leads to *Coulomb's law.* Feynman now instructs us that we can combine any line for a propagating electron (or when it travels backwards, the positron) and any line for a propagating photon tied together with the vertex where an electron line emits a photon to make up new diagrams. Every other diagram differing from the one above constitutes quantum corrections to the basic force. It was through the work of the three scientists above that it was shown that every such diagram can be made to make sense to give finite answers. It is said that *QED* is *renormalisable.* The strength of the force as in *Coulomb's law* is governed by the magnitude of the vertex which is the electric charge *e* in QED and for the diagram above it is proportional to the square of *e* and is the *Fine Structure Constant* = 1/137. Since this is a small number it makes sense to write the amplitude in a series of terms with higher and higher powers of since that factor will be smaller and smaller for ever increasing complexity of the diagram. The higher order terms are higher quantum corrections and the *perturbation expansion* that we have defined will have smaller and smaller terms as we go to higher quantum corrections.

4. Nuclear Forces

Since there were only two basic forces known in the beginning of the 20th century, gravitation and electromagnetism, and it was seen that electromagnetism is responsible for the forces in the atom, it was natural to believe that it was also responsible for the forces keeping the nucleus together. In the 1920's it was known that the nuclei contain protons, in fact the hydrogen nucleus is just a proton, and somehow it was believed that electrons could be involved in keeping the protons together. However, an idea like this has immediate problems. What is the difference between the electrons in the nucleus and the ones in orbit around the nucleus? What is the consequence of Heisenberg's uncertainty relation if electrons are squeezed into the small nucleus? The only support for the idea, apart from there being no other known elementary particles, was that in certain radioactive decays electrons were seen to come from the nucleus. However, in 1932 James Chadwick discovered a new type of radiation that could emanate from the nuclei, a neutral one and his experiment showed that there are indeed electrically neutral particles inside the nuclei, which came to be called neutrons. Soon after Eugene Wigner explained the nuclei as a consequence of two different nuclear forces. The Strong Nuclear Force is an attractive force between protons and neutrons that keep the nucleus together and the Weak Nuclear Force is responsible for the radioactive decay of certain nuclei. It was realized that the strength of the two forces differed a lot. The typical ratio is of the order of 1014 at ordinary energies.

5. Strong Interactions

A natural idea now was to search for a mechanism like the one in electromagnetism to mediate the strong force. Already in 1935 Hideki Yukawa proposed a field theory for the strong interaction where the mediating field particle was to be called a meson.

However, there is a significant difference between the strong force and the electromagnetic one in that the strong force has a very short range (typically the nuclear radius). This is the reason why it has no classical counterpart and hence had not been discovered in classical physics. Yukawa solved this problem by letting the meson have a mass. Such a particle was also subsequently seemingly found from cosmic rays by Carl Anderson. The discovery of nuclear fission in the late 1930's led to an enormous interest in nuclear physics and in the war years most physicists worked on problems with fission so it was not until after the war that Yukawa's ideas were taken up again. It was then realized that the particle found by Anderson could not be the meson of strong interactions, since it interacted far too little with matter, and it was then shown that this particle, now called the muon, is a heavy cousin of the electron. However, the meson, now called pion, was finally discovered in cosmic rays by Cecil Powell in 1947 and its properties were measured. A new dilemma now appeared. When the big accelerators started to operate in the 1950's, the pions were produced vindicating Yukawa's theory, but when his field theory was scrutinised according to the rules set up by Feynman, it was shown that indeed the theory is renormalisable but the coupling constant is huge, larger than one. This means that a diagram with several interactions will give a larger contribution than the naive one with the exchange of only one pion, which is the one though that does gives a rough picture of the scattering of two protons.

The perturbation expansion does not make sense. Also the scattering of protons produced new strongly interacting particles beside the pion, which were named hadrons. Indeed a huge menagerie of elementary particles were discovered, some of them with a life time of some 10^{-8} to 10^{-10} s and some with a lifetime of 10^{-23} s. This problem was solved by Murray Gell-Mann when he proposed that all the strongly interacting particles are indeed bound states of even more fundamental states, the *quarks.* This idea was eventually experimentally verified in the Stanford experiments in the years around 1970 led by Jerome Friedman, Henry Kendall and Richard Taylor. To understand the forces inside the nucleus one really had to understand the field theory for quarks. Before describing the forces between quarks we have to discuss the other nuclear force, the weak one.

6. Weak Interactions

In 1896 Henri Becquerel discovered that uranium salts emit a radiation; they are radioactive. His work was followed up by Marie and Pierre Curie who discovered that several atoms disintegrated by sending out radioactivity. With the discovery of the neutron it was realized that this phenomenon is another aspect of a force at work. It was found that the neutron decays into a proton and an electron and a then hypothetical particle proposed by Wolfgang Pauli, which came to be called the neutrino (really the antineutrino). Since in the nucleus the mass of the nucleons are virtual the process can also go the other way in which a proton decays into a neutron, a positron and a neutrino. The first to set up a model for this interaction was Enrico Fermi in which it was supposed that the interaction was instantaneous among the matter particles. In the late 1950s Fermi's theory was modified to account for parity violation by Marshak and Sudarshan and by Feynman and Gell-Mann. Parity violation of the weak interactions had been postulated by Tsung-Dao Lee and Chen Ning Yang in 1956 and experimentally verified by Wu and collaborators the year after. (The weak interactions can distinguish between left and right.)

However, the model introduced had severe problems. It is not renormalisable so it cannot really make sense as a general theory. On the other hand the model worked extremely well for many processes. How could one reconcile these two facts? During the 1960's new field theoretic descriptions were proposed and to reconcile the facts above one introduced mediating particles that were extremely heavy. For low energy processes such a particle can only propagate a very short distance and in practice it will look as if the interaction takes place in one point giving the model above for the energies that at the time could be probed. The scheme used, the so-called Non-Abelian Gauge Theories' were used by Sheldon Glashow, Steven Weinberg and Abdus Salam in independent works to suggest a model that would generalise the model above. Such a field theory is a generalisation of QED in which there are several mediating particles which also can have self interactions. In the beginning of the 1970's this scheme of models were proven to be renormalisable and hence good quantum theories by Gerhard tHooft and Tini Veltman. Overwhelming experimental evidence for the model was gathered in the 1970's and finally in 1983 the mediating particles were discovered at CERN in an experiment led by Carlo Rubbia and Simon van der Meer. Indeed the mediating particles are very heavy, almost 100 times the mass of the proton.

7. Theory for Strong Interactions

A remarkable feature of the SLAC experiments that verified the existence of quarks was 'scaling'. The cross sections for the deep inelastic scattering of electrons on protons depended on fewer kinematical variables for higher energies. The cross sections scaled. This phenomenon was theoretically suggested by James Bjorken and the data showed it clearly. Richard Feynman explained it by assuming that the protons consisted of point-like constituents. To explain scaling these constituents must have a coupling strength that decreases with energy, opposite to the case of QED. This was called 'asymptotic freedom'. It was quite difficult to believe that a quantum field theory could be asymptotically free since the energy dependence of the coupling constant is due to the screening from pairs of virtual particles. Relativistic quantum mechanics allow for such pairs if they do not live too long. This is due to Heisenberg's uncertainty principle and the fact that energy is the same as mass according to Einstein's famous formula.

Asymptotic freedom must mean that the quark charges are antiscreened, which as said was hard to believe to exist in a quantum field theory. However, in 1973, David Gross, David Politzer and Frank Wilczek simultaneously found that for a non-abelian gauge field theory the requirement of asymptotic freedom is satisfied if there are not too many quarks. The key to the solution was that the vector particles mediating the force, the gluons, do indeed antiscreen. This can be understood since the charges of the quarks and the gluons, the "colour charges" satisfy more complicated relations than the simpler electric charges. There are three different colours and their anticolours. While the quarks have a colour charge, the gluons have a colour and an anticolour charge. Hence virtual gluons can line up with charges screening each other while the strength of the field increases. The discovery of asymptotic freedom opened up for a non-abelian gauge field theory for the interactions among quarks and it was called QuantumChromodynamics, QCD. Over the years this theory has been very successfully tested at the large accelerators and it is now solidly established as the theory of the strong interactions.

8. The Standard Model

The success of non-abelian gauge theories showed that all the interactions could be unified in a common framework. This led to the so-called Standard Model in which all the matter particles are treated together, i.e. the electron and its heavier partners the muon and the tau-particle and the corresponding neutrinos, which all have only weak interactions, together with the quarks which can have both strong and weak interactions. The force particles, i.e. the mediators, are then the photon for electromagnetism, the *W* and *Z* particles for the weak force and the gluons for the strong force. Even though the Standard Model unifies the interactions there are differences in the details. The photon and the gluons are massless particles while the *W* and *Z* particles have a mass. The *photon* leads to Coulomb's law for large distances while the *gluons* lead to a confining force between the quarks. This is in fact due to the asymptotic freedom, which can also be interpreted to say that the coupling strength increases with lower energy, which quantum mechanically also means that it increases with distance. In fact this increase is like the one for a spring, such that the quarks are permanently bound in the hadrons. Even so the properties of the gluons have been firmly established by experimenters.

9. Unification of all Interactions

In the standard model above there is no mentioning of the gravitational force. It has been said that it is so tremendously weak that we do not need to take it into account at particle experiments. However, on general grounds there must be a quantum version of the gravity force that acts at small enough distances. If we try to just copy the quantisation of the electromagnetic field in terms of photons we should quantize the gravity field into so-called *gravitons.* However, the procedure of Feynman, Tomonaga and Schwinger does not work here. Einstein's gravity is non-renormalisable. Where is the problem? Is it Einstein's theory or quantum mechanics that is not complete? The two great conceptual milestones of the 20th century, Quantum Mechanics and Einstein's General Relativity are simply not consistent with each other. Einstein thought for his whole life that quantum mechanics is indeed incomplete, but so many tests of it have by now been made that physicists are instead trying to generalise Einstein's theory. The remarkable success with the Standard Model has also shown that the idea of unification of the forces is a valid one. Why are there four different forces or are they really different? They do indeed, show up as different forces in the experiments we do, but the Standard Model shows that the electromagnetic and the weak forces are unified for energies above 100 GeV. Similarly the model shows that also the strong force seemingly so different unifies with the other one at energies above 10^{15}GeV. Can the gravitational one be fit into this scheme?

It can be shown that at energies of the order of 10^{19} GeV the gravity force will be as strong as the other ones, so there should be a unification of all the forces at least at that energy, which is an energy so unbelievably high that it has only occurred in our universe at a time 10^{-42} s after the Big Bang. However, physics should also be able to describe phenomena that occurred then, so there should be a unified picture which also includes gravity. Such a scheme has now been proposed, *The Superstring Model* in which particles are described by one-dimensional objects, strings. This model indeed gives Einstein's theory for low energies and can be made compatible with the Standard Model at the energies where it has been probed. It is also a finite quantum theory so a perturbation theory for gravity based on the

Superstring Model is indeed consistent. It is still too early to say if this is the final 'theory of everything', but there is no paradox or inconsistency in the model as far as has been understood. Finally the model makes one more unification, namely of the matter particles and the force particles, having just one sort of particles. This is also the ultimate goal of physicists, to have one unified force and one unified kind of particles.

In Section 3 of this course you will cover these topics:

- Systems Of Forces And Moments
- Objects In Equilibrium

Topic Objective:

At the end of this topic student would be able to:

- Define the term force
- Describe the normal force
- Highlight the non fundamental force

Definition/Overview:

Force: a force is that which can cause an object with mass to change its velocity. Force has both magnitude and direction, making it a vector quantity. Newton's second law states that an object with a constant mass will accelerate in proportion to the net force acting upon and in inverse proportion to its mass. Equivalently, the net force on an object equals the rate at which its momentum changes

Key Points:

1. Force

Forces acting on three-dimensional objects may also cause them to rotate or deform, or result in a change in pressure or even change volume in some cases. The tendency of a force to cause changes in rotational speed about an axis is called torque. Deformation and pressure are the result of stress forces within an object. Since antiquity, scientists have used the concept of force in the study of stationary and moving objects. The study of forces advanced with descriptions made by the third century BC philosopher Archimedes of how forces interact in simple machines. Prior to this, descriptions of forces by Aristotle incorporated fundamental misunderstandings. By the seventeenth century, Sir Issac Newton corrected these misunderstandings with mathematical insight that remained unchanged for nearly three hundred years.

By the early 20th century, Einstein in his theory of general relativity successfully predicted the failure of Newton's model for gravity by ushering in the concept of a space-time continuum. The recent theory of particle physics known as the Standard Model associate forces at the level of quantum mechanics. The Standard Model predicts that exchange particles called gauge bosons are the fundamental means by which forces are emitted and absorbed. Only four main interactions are known: in order of decreasing strength, they are: strong, electromagnetic, weak, and gravitational. High-energy particle physics observations made during the 1970s and 1980s confirmed that the weak and electromagnetic forces are expressions of a more fundamental electroweak interaction

2. Non-fundamental forces

Some forces are consequences of fundamental. In such situations, idealized models can be utilized to gain physical insight.

3. Normal force

The normal force is the repulsive force of interaction between atoms at close contact. When their electron clouds overlap, Pauli repulsion (due to fermionic nature of electrons) follows resulting in the force which acts normal to the surface interface between two objects. The normal force, for example, is responsible for the structural integrity of tables and floors as well as being the force that responds whenever an external force pushes on a solid object. An example of the normal force in action is the impact force on an object crashing into an immobile surface

4. Friction

Friction is a surface force that opposes motion. The frictional force is directly related to the normal force which acts to keep two solid objects separated at the point of contact. There are two broad classifications of frictional forces: static friction and kinetic friction. The static friction force (F_{sf}) will exactly oppose forces applied to an object parallel to a surface contact up to the limit specified by the coefficient of static friction (μ_{sf}) multiplied by the normal force (F_N). In other words the magnitude of the static friction force satisfies the inequality:

.

The kinetic friction force (F_{kf}) is independent of both the forces applied and the movement of the object. Thus, the magnitude of the force equals:

$$F_{kf} = \mu_{kf} F_N,$$

where μ_{kf} is the coefficient of kinetic friction. For most surface interfaces, the coefficient of kinetic friction is less than the coefficient of static friction

5. Continuum mechanics

Newton's laws and Newtonian mechanics in general were first developed to describe how forces affect idealized point particles rather than three-dimensional objects. However, in real life, matter has extended structure and forces that act on one part of an object might affect other parts of an object. For situations where lattice holding together the atoms in an object is able to flow, contract, expand, or otherwise change shape, the theories of continuum mechanics describe the way forces affect the material. For example, in extended fluids, differences in pressure result in forces being directed along the pressure gradients as follows:

Where, *V* is the volume of the object in the fluid and *P* is the scalar function that describes the pressure at all locations in space. Pressure gradients and differentials result in the buoyant force for fluids suspended in gravitational fields, winds in atmospheric science, and the lift associated with aerodynamics and flight. A specific instance of such a force that is associated with dynamic pressure is fluid resistance: a body force that resists the motion of an object through a fluid due to viscosity. For so-called "Stokes' drag" the force is approximately proportional to the velocity, but opposite in direction:

Where,

b is a constant that depends on the properties of the fluid and the dimensions of the object (usually the cross-sectional area), and

is the velocity of the object.

More formally, forces in continuum mechanics are fully described by a stress tensor with terms that are roughly defined as

Where. *A* is the relevant cross-sectional area for the volume for which the stress-tensor is being calculated. This formalism includes pressure terms associated with forces that act normal to the cross-sectional area (the matrix diagonals of the tensor) as well as shear terms associated with forces that act parallel to the cross-sectional area (the off-diagonal elements). The stress tensor accounts for forces that cause all deformations including also tensile stresses and compressions.

When the drag force (F_d) associated with air resistance becomes equal in magnitude to the force of gravity on a falling object (F_g), the object reaches a state of dynamical equilibrium at terminal velocity.

6. Moments

"Principle of Moments" redirects here. For the Robert Plant album, see The Principle of Moments. For a more abstract concept of moments that evolved from this concept of physics. The term "moment" can refer to many different concepts:

- **Moment of force** (often just *moment*) is a synonym for torque, an important basic concept in physics, civil engineering, and mechanical engineering. In the context of mechanical engineering, the terms are not necessarily interchangeable, but one or the other may be preferred in a specific context. For example, "torque" is usually used to describe a rotational force down a shaft, for example a turning screw-driver, whereas "moment" is more often used to describe a bending force on a beam.
    - **Moment arm** is a quantity used when calculating torque. See the article torque.
    - The **Principle of moments** is a theorem concerning torques. See the article torque.
- **Moment of a vector** is a generalization of the moment of force. The moment **M** of a vector **B** about the point **A** is

  where

  is the vector from point A to the position where quantity **B** is applied.

  represents the cross product of the vectors.

  Thus **M** can be referred to as "the moment **M** with respect to the axis that goes through the point *A*", or simply "the moment **M** around *A*". If *A* is the origin, or, informally, if the axis involved is clear from context, one often omits *A* and says simply *moment*.

  When **B** is the force, the moment of force is the torque as defined above.

- **Moment of inertia** ( ) is analogous to mass in discussions of rotational motion.

- **Angular momentum** ($L = I\omega$) is the rotational analog of momentum. (Historically, angular momentum was sometimes referred to as "moment of momentum".)
- **Magnetic moment** ( ) is a dipole moment measuring the strength and direction of a magnetic source.

Simply put, The equation of the moment of the force. **M=Fd**

M= moment F=force d= distance from the pivot

Topic Objective:

At the end of this topic student would be able to:

- Define the term equilibrium
- Describe the objects in equilibrium
- Highlight the equilibrium mode distribution

Definition/Overview:

Equilibrium: Equilibrium is:

- Equality of weight or force; equipoise or a state of rest produced by the mutual counteraction of two or more forces.
- A level position; a just poise or balance in respect to an object, so that it remains firm; equipoise; as, to preserve the equilibrium of the body
- A balancing of the mind between motives or reasons, with consequent indecision and doubt

Key Points:

1. Objects in Equilibrium

A standard definition of **static equilibrium** is:

- A system of particles is in static equilibrium when all the particles of the system are at rest and the total force on each particle is permanently zero

This is a strict definition, and often the term "static equilibrium" is used in a more relaxed manner interchangeably with "mechanical equilibrium", as a standard definition of **mechanical equilibrium** for a particle is:

- The necessary and sufficient conditions for a particle to be in mechanical equilibrium is that the net force acting upon the particle is zero

The necessary conditions for **mechanical equilibrium** for a system of particles are:

- The vector sum of all external forces is zero;
- The sum of the moments of all *external forces* about any line is zero

As applied to a rigid body, the necessary and sufficient conditions become a rigid body is in mechanical equilibrium when the sum of all forces on all particles of the system is zero, and also the sum of all torques on all particles of the system is zero. A rigid body in mechanical equilibrium is undergoing neither linear nor rotational acceleration; however it could be translating or rotating at a constant velocity. However, this definition is of little use in continuum mechanics, for which the idea of a particle is foreign. In addition, this definition gives no information as to one of the most important and interesting aspects of equilibrium states their stability. An alternative definition of equilibrium that applies to conservative systems and often proves more useful is a system is in mechanical equilibrium if its position in configuration space is a point at which the gradient with respect to the generalized coordinates of the potential energy is zero.

Because of the fundamental relationship between force and energy, this definition is equivalent to the first definition. However, the definition involving energy can be readily extended to yield information about the stability of the equilibrium state. For example, from elementary calculus, we know that a necessary condition for a local minimum *or* a maximum of a differentiable function is a vanishing first derivative (that is, the first derivative is becoming zero). To determine whether a point is a minimum or maximum, one may be able to use the second derivative test. The consequences to the stability of the equilibrium state are as follows:

- **Second derivative < 0 :** The potential energy is at a local maximum, which means that the system is in an unstable equilibrium state. If the system is displaced an arbitrarily small distance from the equilibrium state, the forces of the system cause it to move even farther away.
- **Second derivative > 0 :** The potential energy is at a local minimum. This is a stable equilibrium. The response to a small perturbation is forces that tend to restore the equilibrium. If more than one stable equilibrium state is possible for a system, any equilibria whose potential energy is higher than the absolute minimum represent metastable states.
- **Second derivative = 0** or does not exist: The second derivative test fails, and one must typically resort to using the first derivative test. Both of the previous results are still possible, as is a third: this could be a region in which the energy does not vary, in which case the equilibrium is called neutral or indifferent or marginally stable. To lowest order, if the system is displaced a small amount, it will stay in the new state.

In more than one dimension, it is possible to get different results in different directions, for example stability with respect to displacements in the *x*-direction but instability in the *y*-

direction, a case known as a saddle point. Without further qualification, an equilibrium is stable only if it is stable in all directions. The special case of mechanical equilibrium of a stationary object is **static equilibrium**. A paperweight on a desk would be in static equilibrium. The minimal number of static equilibria of homogeneous, convex bodies (when resting under gravity on a horizontal surface) is of special interest. In the planar case, the minimal number is 4, while in three dimensions one can build an object with just one stable and one unstable balance point, this is called Gomboc. A child sliding down a slide at constant speed would be in mechanical equilibrium, but not in static equilibrium.

2. Equilibrium mode distribution

The **equilibrium mode [power] distribution** of light travelling in an optical waveguide or fiber is the distribution of light that is no longer changing with fibre length or with input modal excitation. This phenomenon requires both mode filtering and mode mixing to occur in the fibre to produce a state that is independent of the mode power distribution launched by the light source. At propagation distances exceeding the equilibrium length, intramodal pulse distortion increases (bandwidth decreases) as the square root of length. The term ***equilibrium length*** is sometimes used to describe a *stationary* mode distribution, which is the length of multimode optical fiber necessary to attain a static mode distribution from a specific excitation condition. Equilibrium length is, strictly, the longest such length, as would result from a widely variable range of input excitation. Other terms for equilibrium length are **equilibrium coupling length** and **equilibrium mode distribution length**.

Equilibrium mode [power] distributions were reported in early multimode transmission systems at propagation distances as short as a few hundred metres. However, as fibre manufacturing improved, the minute waveguide dimensional and structural changes that produce mode-mixing have been greatly reduced. The length of fibre required to attain true equilibrium is now much greater than the length of practical multimode transmission systems, which makes the term effectively obsolete. In the absence of strong mode-mixing, high order mode filtering is the primary remaining mechanism for potential change of an input mode power distribution. If a well-aligned laser is the optical source, the mode power distribution is highly concentrated in the lowest order modes, and remains essentially unchanged with distance due to the lack of mode-mixing. If an optical source that overfills the fibre is used, only the highest order guided mode group experiences excess attenuation, and the mode power distribution becomes slightly filtered as a result. (Mandrel wrapping is a viable method to artificially create this state.) Such mode power distributions are stationary; neither changes with fibre length, but equilibrium does not exist in either case because the distributions remain dependent on the input power distribution.

Example/Case Study:

Example 1

The first example will make use of the hinged rod supported by a rope, as discussed above. The rod has a mass of 1.4 kg, and there is an angle of 34 between the rope and the rod.

- What is the tension in the rope?
- What are the two components of the support force exerted by the hinge?

The free-body diagram is shown below, with the support force provided by the hinge split up into x and y components. If you aren't sure which way such forces go, simply guess, and if you guess wrong you'll just get a negative sign for that force.

Something we'll assume in this example is that the rod is uniform, so the weight acts at the center of the rod (the center is the center of mass, in other words). As usual, sum the forces in the x and y directions:

There are too many unknowns here, and this is why summing the torques can be so useful. To sum torques, choose a point to take torques around; a sensible point to choose is one that one or two unknown forces go through, because they will nt appear in the torque equation. In this case, choosing the hinge as the point to take torques around eliminated both components of the support force at the hinge. As with forces, where you choose plus and minus directions, choose a positive and negative direction for torques. In this case, let's make counter-clockwise negative and clockwise positive.

This can be solved for T, the tension in the rope. Note that r, which represents the length of the rod, cancels out:

This can be substituted back into the force equations to find the components of the hinge force:

Example 2 - a step-ladder

A step-ladder stands on a frictionless horizontal surface, with just the crossbar keeping the ladder standing. The mass is 20 kg; what is the tension in the crossbar?

This is something of a tricky problem, because you have to draw the free-body diagram of the entire ladder to figure out the normal forces, and then draw the free-body diagram of one half of the ladder to complete the solution. This is also what makes it a good example to look at, however.

Consider first the free-body diagram of the entire ladder. The floor is frictionless, so there are no horizontal forces exerted by the floor. The ladder is uniform, so the weight acts at the center of mass, which is halfway up the ladder and halfway between the two legs. Summing forces in the y-direction gives:

One way to approach this is to say that the ladder is symmetric, and there is no reason for the two normal forces to be different; each one should be equal to half the weight of the ladder. If you don't like this argument, simply take torques about one of the points where the ladder touches the floor. This will give you an equation saying that one normal force is equal to half the ladder's weight, so the other normal force must be equal to half the weight, too. Either way, you should be able to show that:

Now consider the free-body diagram of the left-hand side of the ladder. I'll attach a 1/2 as a subscript to the mass, to remind us that the mass of half the ladder is half the mass of the entire ladder.

Taking torques around the top of the ladder eliminates the unknown contact force (F) coming from the other half of the ladder, and gives (this time taking clockwise to be positive):

This can be solved to find the tension in the crossbar:

In Section 4 of this course you will cover these topics:

- Structures In Equilibrium
- Centroids And Centers Of Mass 316

Topic Objective:

At the end of this topic student would be able to:

- Define the term equilibrium
- Describe the equilibrium in structures and mathematical consideration
- Highlight the application of equilibrium

Definition/Overview:

Equilibrium: Equilibrium is the condition of a system in which competing influences are balanced

Key Points:

1. Equilibrium in Structures: Mathematical consideration

For a volume of a fluid which is not in motion or is in a state of constant motion, Newton's Laws state that it must have zero net force on it the forces up must equal the forces down. This force balance is called the hydrostatic balance. We can split the gas into a large number of cuboid volume elements. By considering just one element, we can work out what happens to the gas as a whole. There are 3 forces: The force downwards onto the top of the cuboid from the pressure, P, of the fluid above it is, from the definition of pressure, Similarly, the force on the volume element from the pressure of the fluid below pushing upwards is

In this equation, the minus sign comes from the direction this force supports the volume element, rather than pull it down (We are presuming that positive force acts down, however this is irrelevant) Finally, the weight of the volume element causes a force downwards. If the density is ρ, the volume is V and g the standard gravity, then:

We can split volume into the area of the top or bottom, times the height.

By balancing these forces, the total force on the gas is

This is zero if the gas isn't moving. If we divide by A,

$P_{top} - P_{bottom}$ is a change in pressure, and h is the height of the volume element a change in the distance above the ground. By saying these changes are infinitesimally small, the equation can be written in differential form.

Density changes with pressure, and gravity changes with height, so the equation would be:

Finally that this last equation can be derived by solving the three-dimensional Navier-Stokes equations for the equilibrium situation where

Then the only non-trivial equation is the *z*-equation, which now reads

Thus, hydrostatic balance can be regarded as a particularly simple equilibrium solution of the Navier-Stokes equations.

2. Applications

2.1. Fluids

The hydrostatic equilibrium pertains to hydrostatics and the principles of equilibrium of fluids. A hydrostatic balance is a particular balance for weighing substances in water. Hydrostatic balance allows the discovery of their specific gravities.

2.2. Astrophysics

Hydrostatic equilibrium is the reason stars don't implode, or explode. In astrophysics, in any given layer of a star, there is a balance between the thermal pressure (outward) and the weight of the material above pressing downward (inward). This balance is called hydrostatic equilibrium. A star is like a balloon. In a balloon, the gas inside the balloon pushes outward and the Earth's atmospheric pressure plus the elastic material supply just enough inward compression to balance the gas pressure. In the case of a star, the star's internal gravity supplies the inward compression. The isotropic gravitational field compresses the star into the most compact shape possible: a sphere.

However that a star becomes a sphere only in the ideal case where only its own self-gravity is involved. In real situations there are other forces at play that alter the outcome, most notably centrifugal force from a star's rotation. A rotating star in hydrostatic equilibrium is an oblate spheroid up to a certain angular velocity; above that point it becomes a Jacobi (scalene) ellipsoid, and at still higher rotations piriform. An extreme example of this is the star Vega, which has a rotation period of 12.5 hours and is about 20% fatter at the equator than at the poles because of it. If the star has a massive nearby companion object then tidal forces come into play as well, distorting the star into an ellipsoidal shape when rotation alone would make it a spheroid. An example of this is Beta Lyrae. It is also important for the intracluster medium, where it restricts the amount of gas that can be present in the core of a cluster of galaxies.

2.3. Planetary geology

The concept of hydrostatic equilibrium has also become important in determining whether an astronomical object is a planet, dwarf planet, or small solar system body. According to the definition of planet adopted by the International Astronomical Union in 2006, planets and dwarf planets are objects that have sufficient gravity to overcome their own rigidity and assume hydrostatic equilibrium. Sometimes this means a spheroid. However, in the cases of moons in synchronous orbit, tidal forces create a scalene ellipsoidal shape, and the quickly rotating dwarf planet Haumea also appears to be scalene.

Since the terrestrial planets and dwarf planets (and likewise the larger satellites, like the Moon and Io) have rough surfaces and so are not in perfect equilibrium, this definition evidently has some flexibility, but as of yet a specific means of quantifying an object's shape by this standard has not been announced. The amount of leeway afforded the definition could affect the classification of the asteroid Vesta, which appears to have solidified while in hydrostatic equilibrium but to have subsequently been significantly deformed by a large impact.

2.4. Atmospherics

Hydrostatic equilibrium can explain why the Earth's atmosphere does not collapse to a very thin layer on the ground. In the atmosphere, the pressure of air decreases with increasing altitude. This causes an upward force, called the pressure gradient force, which tries to smooth over pressure differences. The force of gravity, on the other hand, almost exactly balances this out, keeping the atmosphere bound to the earth and maintaining pressure differences with altitude. Without the pressure gradient force, the atmosphere would collapse to a much thinner shell around the earth, and without the force of gravity, the pressure gradient force would diffuse the atmosphere into space, leaving Earth with hardly any atmosphere.

Topic Objective:

At the end of this topic student would be able to:

- Define the term centroid
- Describe the properties of centroid
- Highlight the integral formula for centroid

Definition/Overview:

Centroid: the centroid, geometric center, or barycenter of a plane figure X is the intersection of all straight lines that divide X into two parts of equal moment about the line. Informally, it is the "average" of all points of X. The definition extends to any object X in n-dimensional space: its centroid is the intersection of all hyperplanes that divide X into two parts of equal moment.

Key Points:

1. Centroid

The word centroid may mean the geometric center of the object's shape, as above; or its physical barycenter, which is its center of mass or the center of gravity, depending on the context. Informally, the barycenter is the average of all points, weighted by the local density or specific weight, respectively.

2. Properties

The geometric centroid of a convex object always lies in the object. It is the point of concurrency of the triangle's medians. A non-convex object might have a centroid that is outside the figure itself. The centroid of a ring or a bowl, for example, lies in the object's central void.

If the centroid is defined, it is a fixed point of all isometries in its symmetry group. In particular, the geometric centroid of an object lies in the intersection of all its hyperplanes of symmetry. The centroid of many figures (regular polygon, regular polyhedron, cylinder, rectangle, rhombus, circle, sphere, ellipse, ellipsoid, superellipse, superellipsoid, etc.) can be determined by this principle alone. For the same reason, the centroid of an object with translational symmetry is undefined (or lies outside the enclosing space), because a translation has no fixed point.

3. Centroid of a finite set of points

The centroid of a finite set of points in

is

4. Centroid by geometric decomposition

The centroid of a plane figure *X* can be computed by dividing it into a finite number of simpler figures , computing the centroid C_i and area A_i of each part, and then computing

The same formula holds for any three-dimensional objects, except that each A_i should be the volume of X_i, rather than its area. It also holds for any subset of , for any dimension *d*, with the areas replaced by the *d*-dimensional measures of the parts.

This formula holds even if the parts overlap and/or extend outside the set *X*, provided that the measures A_i are taken with positive and negative signs in such a way that the sum of the A_i of all parts that enclose a given point *x* is 1 if *x* belongs to *X*, and 0 otherwise.

5. Integral formula

The centroid of a subset *X* of can also be computed by the integral

Where, the integral is taken over the whole space , and f is the characteristic function of the subset, which is 1 inside X and 0 outside it. (However, this formula cannot be applied if the object has zero measure, or if either integral diverges.)

6. Position of the CM using polar coordinates. (Center of mass).

Where, σ denotes the surface mass density (mass/area) of the sheet and *dA* is the element of area $dA = r dr d\varphi$.

7. Centroid of triangle and tetrahedron

The centroid of a triangle is the point of intersection of its medians (the lines joining each vertex with the midpoint of the opposite side). The centroid divides each of the medians in the ratio 2:1, which is to say it is located ⅓ of the perpendicular distance between each side and the opposing point. (As illustrated in the figures to the right). The centroid is the triangle's center of mass if the triangle is made from a uniform sheet of material. Its Cartesian coordinates are the means of the coordinates of the three vertices. That is, if the three vertices are $a = (x_a, y_a)$, $b = (x_b, y_b)$, and $c = (x_c, y_c)$, then the centroid is

A similar result holds for a tetrahedron: its centroid is the intersection of all line segments that connect each vertex to the centroid of the opposite face. These line segments are divided by the centroid in the ratio 3:1. The result generalizes to any *n*-dimensional simplex in the obvious way. If the set of vertices of a simplex is $v_0,...,v_n$, then considering the vertices as vectors, the centroid is

The isogonal conjugate of a triangle's centroid is its symmedian point.

8. Centroid of polygon

The centroid of a non-overlapping closed polygon defined by *n* vertices (x_i , y_i) can be calculated as follows. The area of the polygon is

and its centroid is C = (Cx,Cy) where

In these formulas, the vertex (x_n , y_n) is assumed to be the same as (x_0 , y_0).

In Section 5 of this course you will cover these topics:

- Moments Of Inertia
- Friction
- Internal Forces And Moments
- Virtual Work And Potential Energy

Topic Objective:

At the end of this topic student would be able to:

- Define the term moments of inertia
- Describe the scalar moment of inertia

Definition/Overview:

Moments of Inertia: **Moment of inertia**, also called **mass moment of inertia** or the **angular mass**, (SI units kg m^2, Imperial Unit slug ft^2) is a measure of an object's resistance to changes in its rotation rate. It is the rotational analog of mass. That is, it is the inertia of a rigid rotating body with respect to its rotation. The moment of inertia plays much the same role in rotational dynamics as mass does in basic dynamics, determining the relationship between angular momentum and angular velocity, torque and angular acceleration, and several other quantities. While a simple scalar treatment of the moment of inertia suffices for many situations, a more advanced tensor treatment allows the analysis of such complicated systems as spinning tops and gyroscope motion.

Key Points:

1. Moments of Inertia

The moment of inertia of an object about a given axis describes how difficult it is to change its angular motion about that axis. For example, consider two discs (A and B) of the same mass. Disc A has a larger radius than disc B. Assuming that there is uniform thickness and mass distribution, it requires more effort to accelerate disc A (change its angular velocity) because its mass is distributed further from its axis of rotation: mass that is further out from that axis must, for a given angular velocity, move more quickly than mass closer in. In this case, disc A has a larger moment of inertia than disc B. The moment of inertia of an object can change if its shape changes. A figure skater who begins a spin with arms outstretched provides a striking example. By pulling in her arms, she reduces her moment of inertia, causing her to spin faster (by the conservation of angular momentum).

The moment of inertia has two forms, a scalar form I (used when the axis of rotation is known) and a more general tensor form that does not require knowing the axis of rotation. The scalar moment of inertia I (often called simply the "moment of inertia") allows a succinct analysis of many simple problems in rotational dynamics, such as objects rolling down inclines and the behavior of pulleys. For instance, while a block of any shape will slide down a frictionless decline at the same rate, rolling objects may descend at different rates, depending on their moments of inertia. A hoop will descend more slowly than a solid disk of equal mass and radius because more of its mass is located far from the axis of rotation, and thus needs to move faster if the hoop rolls at the same angular velocity. However, for (more complicated) problems in which the axis of rotation can change, the scalar treatment is inadequate, and the tensor treatment must be used (although shortcuts are possible in special situations). Examples requiring such a treatment include gyroscopes, tops, and even satellites, all objects whose alignment can change. The moment of inertia can also be called the mass moment of inertia (especially by mechanical engineers) to avoid confusion with the second moment of area, which is sometimes called the moment of inertia (especially by structural engineers) and denoted by the same symbol I. The easiest way to differentiate these quantities is through their units. In addition, the moment of inertia should not be confused with the polar moment of inertia, which is a measure of an object's ability to resist torsion (twisting).

2. Scalar moment of inertia

A simple definition of the **moment of inertia** of any object, be it a point mass or a 3D-structure, is given by:

Where,

'dm' is the mass of an infinitesimally small part of the body

and *r* is the (perpendicular) distance of the point mass to the axis of rotation.

3. Detailed Analysis

The (scalar) moment of inertia of a point mass rotating about a known axis is defined by

The moment of inertia is additive. Thus, for a rigid body consisting of *N* point masses m_i with distances r_i to the rotation axis, the total moment of inertia equals the sum of the point-mass moments of inertia:

For a solid body described by a continuous mass density function ρ(**r**), the moment of inertia about a known axis can be calculated by integrating the square of the distance (weighted by the mass density) from a point in the body to the rotation axis:

Where,

V is the volume occupied by the object.

ρ is the spatial density function of the object, and

are coordinates of a point inside the body.

Here k is 1/2 and r is the radius used in determining the moment. Based on dimensional analysis alone, the moment of inertia of a non-point object must take the form:

Where,

M is the mass

R is the radius of the object from the center of mass (in some cases, the length of the object is used instead.)

k is a dimensionless constant called the *inertia constant* that varies with the object in consideration.

Inertial constants are used to account for the differences in the placement of the mass from the center of rotation. Examples include:

- *k* = 1, thin ring or thin-walled cylinder around its center,
- *k* = 2/5, solid sphere around its center
- *k* = 1/2, solid cylinder or disk around its center.

4. Parallel axis theorem

Once the moment of inertia has been calculated for rotations about the center of mass of a rigid body, one can conveniently recalculate the moment of inertia for all parallel rotation axes as well, without having to resort to the formal definition. If the axis of rotation is displaced by a distance *R* from the center of mass axis of rotation (e.g. spinning a disc about a point on its periphery, rather than through its center,) the displaced and center-moment of inertia are related as follows:

This theorem is also known as the *parallel axes rule* and is a special case of *Steiner's parallel-axis theorem*.

5. Composite bodies

If a body can be decomposed (either physically or conceptually) into several constituent parts, then the moment of inertia of the body about a given axis is obtained by summing the moments of inertia of each constituent part around the same given axis.

6. Equations involving the moment of inertia

The rotational kinetic energy of a rigid body can be expressed in terms of its moment of inertia. For a system with *N* point masses m_i moving with speeds v_i, the rotational kinetic energy *T* equals

Where, ω is the common angular velocity (in radians per second). The final formula

also holds for a continuous distribution of mass with a generalisation of the above derivation from a discrete summation to an integration. In the special case where the angular momentum vector is parallel to the angular velocity vector, one can relate them by the equation

Where, *L* is the angular momentum and ω is the angular velocity. However, this equation does not hold in many cases of interest, such as the torque-free precession of a rotating object, although its more general tensor form is always correct. When the moment of inertia is constant, one can also relate the torque on an object and its angular acceleration in a similar equation:

Where, τ is the torque and α is the angular acceleration.

Topic Objective:

At the end of this topic student would be able to:

- Define the term friction
- Describe the several varieties of friction
- Highlight the coulomb friction

Definition/Overview:

Friction: Friction is the force resisting the relative lateral (tangential) motion of solid surfaces, fluid layers, or material elements in contact.

Key Points:

1. Friction

Friction is usually subdivided into several varieties:

- **Dry friction** resists relative lateral motion of two solid surfaces in contact. Dry friction is also subdivided into static friction between non-moving surfaces, and kinetic friction (sometimes called sliding friction or dynamic friction) between moving surfaces.
- **Lubricated friction** or fluid friction resists relative lateral motion of two solid surfaces separated by a layer of gas or liquid.
- **Fluid friction** is also used to describe the friction between layers within a fluid that are moving relative to each other.
- **Skin friction** is a component of drag, the force resisting the motion of a solid body through a fluid.
- **Internal friction** is the force resisting motion between the elements making up a solid material while it undergoes deformation.

Friction is not a fundamental force, as it is derived from electromagnetic force between charged particles, including electrons, protons, atoms, and molecules, and so cannot be calculated from first principles, but instead must be found empirically. When contacting surfaces move relative to each other, the friction between the two surfaces converts kinetic energy into thermal energy, or heat. Contrary to earlier explanations, kinetic friction is now understood not to be caused by surface roughness but by chemical bonding between the surfaces. Surface roughness and contact area, however, do affect kinetic friction for micro- and nano-scale objects where surface area forces dominate inertial forces. Friction is distinct from traction. Surface area does not affect friction significantly because as contact area increases, force per unit area decreases. In traction, however, surface area is important.

In the above diagram Arrows are vectors indicating directions and magnitudes of forces. W is the force of weight, N is the normal force, F is an applied force of unidentified type, and Ff is the force of kinetic friction which is equal to the coefficient of kinetic friction times the normal force. Since the magnitude of the applied force is greater than the magnitude of the force of kinetic friction opposing it, the block is moving to the left.

2. Coulomb friction

Coulomb friction, named after Charles-Augustin de Coulomb, is a model used to calculate the force of dry friction. It is governed by the equation:

Where,

- F_f is either the force exerted by friction, or, in the case of equality, the maximum possible magnitude of this force.
- μ is the coefficient of friction, which is an empirical property of the contacting materials,
- F_n is the normal force exerted between the surfaces.

For surfaces at rest relative to each other $\mu = \mu_s$, where μ_s is the *coefficient of static friction*. This is usually larger than its kinetic counterpart. The Coulomb friction may take any value from zero up to F_f, and the direction of the frictional force against a surface is opposite to the motion that surface would experience in the absence of friction. Thus, in the static case, the frictional force is exactly what it must be in order to prevent motion between the surfaces; it balances the net force tending to cause such motion. In this case, rather than providing an estimate of the actual frictional force, the Coulomb approximation provides a threshold value for this force, above which motion would commence.

For surfaces in relative motion $\mu = \mu_k$, where μ_k is the *coefficient of kinetic friction*. The Coulomb friction is equal to F_f, and the frictional force on each surface is exerted in the direction opposite to its motion relative to the other surface. This approximation mathematically follows from the assumptions that surfaces are in atomically close contact only over a small fraction of their overall area, that this contact area is proportional to the normal force (until saturation, which takes place when all area is in atomic contact), and that frictional force is proportional to the applied normal force, independently of the contact area (you can see the experiments on friction from Leonardo Da Vinci). Such reasoning aside, however, the approximation is fundamentally an empirical construction. It is a rule of thumb describing the approximate outcome of an extremely complicated physical interaction. The strength of the approximation is its simplicity and versatility though in general the relationship between normal force and frictional force is not exactly linear (and so the frictional force is not entirely independent of the contact area of the surfaces), the Coulomb approximation is an adequate representation of friction for the analysis of many physical systems.

2.1. Coefficient of friction

The *coefficient of friction* (COF), also known as a *frictional coefficient* or *friction coefficient*, symbolized by the Greek letter μ, is a dimensionless scalar value which describes the ratio of the force of friction between two bodies and the force pressing them together. The coefficient of friction depends on the materials used; for example, ice on steel has a low coefficient of friction, while rubber on pavement has a high coefficient of friction. Coefficients of friction range from near zero to greater than one under good conditions, a tire on concrete may have a coefficient of friction of 1.7. When the surfaces are conjoined, Coulomb friction becomes a very poor approximation (for example, adhesive tape resists sliding even when there is no normal force, or a negative normal force). In this case, the frictional force may depend strongly on the area of contact. Some drag racing tires are adhesive in this way. However, despite the complexity of the fundamental physics behind friction, the relationships are accurate enough to be useful in many applications.

The force of friction is always exerted in a direction that opposes movement (for kinetic friction) or potential movement (for static friction) between the two surfaces. For example, a curling stone sliding along the ice experiences a kinetic force slowing it down. For an example of potential movement, the drive wheels of an accelerating car experience a frictional force pointing forward; if they did not, the wheels would spin, and the rubber would slide backwards along the pavement. Note that it is not the direction of movement of the vehicle they oppose, it is the direction of (potential) sliding between tire and road.

The coefficient of friction is an empirical measurement it has to be measured experimentally, and cannot be found through calculations. Rougher surfaces tend to have higher effective values. Most dry materials in combination have friction coefficient values between 0.3 and 0.6. Values outside this range are rarer, but teflon, for example, can have a coefficient as low as 0.04. A value of zero would mean no friction at all, an elusive property even magnetic levitation vehicles have drag. Rubber in contact with other surfaces can yield friction coefficients from 1 to 2. Occasionally it is maintained that is always < 1, but this is not true. While in most relevant applications < 1, a value above 1 merely implies that the force required to slide an object along the surface is greater than the normal force of the surface on the object. For example, silicone rubber or acrylic rubber-coated surfaces have a coefficient of friction that can be substantially larger than 1. Both static and kinetic coefficients of friction depend on the pair of surfaces in contact; their values are usually approximately determined experimentally. For a given pair of surfaces, the coefficient of static friction is *usually* larger than that of kinetic friction; in some sets the two coefficients are equal, such as teflon-on-teflon.

In the case of kinetic friction, the direction of the friction force may or may not match the direction of motion: a block sliding atop a table with rectilinear motion is subject to friction directed along the line of motion; an automobile making a turn is subject to friction acting perpendicular to the line of motion (in which case it is said to be 'normal' to it). The direction of the static friction force can be visualized as directly opposed to the force that would otherwise cause motion, were it not for the static friction preventing motion. In this case, the friction force exactly cancels the applied force, so the net force given by the vector sum, equals zero. It is important to note that in all cases, Newton's first law of motion holds. While it is often stated that the COF is a "material property," it is better categorized as a "system property." Unlike true material properties (such as conductivity, dielectric constant, yield strength), the COF for any two materials depends on system variables like temperature, velocity, atmosphere and also what are now popularly described as aging and deaging times; as well as on geometric properties of the interface between the materials. For example, a copper pin sliding against a thick copper plate can have a COF that varies from 0.6 at low speeds (metal sliding against metal) to below 0.2 at high speeds when the copper surface begins to melt due to frictional heating. The latter speed, of course, does not determine the COF uniquely; if the pin diameter is increased so that the frictional heating is removed rapidly, the temperature drops, the pin remains solid and the COF rises to that of a 'low speed' test

2.2. The normal force

The normal force is defined as the net force compressing two parallel surfaces together; and its direction is perpendicular to the surfaces. In the simple case of a mass resting on a horizontal surface, the only component of the normal force is the force due to gravity, where N = mg. In this case, the magnitude of the friction force is the product of the mass of the object, the acceleration due to gravity, and the coefficient of friction. However, the coefficient of friction is not a function of mass or volume; it depends only on the material. For instance, a large aluminum block has the same coefficient of friction as a small aluminum block. However, the magnitude of the friction force itself depends on the normal force, and hence the mass of the block.

Materials		Static friction, μ_s	
		Dry & clean	Lubricated
Aluminum	**Steel**	0.61	
Copper	**Steel**	0.53	
Brass	**Steel**	0.51	
Cast iron	**Copper**	1.05	
Cast iron	**Zinc**	0.85	
Concrete (wet)	**Rubber**	0.30	
Concrete (dry)	**Rubber**	1.0	
Copper	**Glass**	0.68	
Glass	**Glass**	0.94	
Polythene	**Steel**	0.2	0.2
Steel	**Steel**	0.80	0.16
Steel	**Teflon**	0.04	0.04
Teflon	**Teflon**	0.04	0.04

[Table 1: Approximate coefficients of friction]

If an object is on a level surface and the force tending to cause it to slide is horizontal, the normal force N between the object and the surface is just its weight, which is equal to its mass multiplied by the acceleration due to earth's gravity, g. If the object is on a tilted surface such as an inclined plane, the normal force is less, because less of the force of gravity is perpendicular to the face of the plane. Therefore, the normal force, and ultimately the frictional force, is determined using vector analysis, usually via a free body diagram. Depending on the situation, the calculation of the normal force may include forces other than gravity.

2.3. Static friction

Static friction is friction between two solid objects that are not moving relative to each other. For example, static friction can prevent an object from sliding down a sloped surface. The coefficient of static friction, typically denoted as μ_s, is usually higher than the coefficient of kinetic friction. The static friction force must be overcome by an applied force before an object can move. The maximum possible friction force between two surfaces before sliding begins is the product of the coefficient of static friction and the normal force: $f = \mu_s F_n$. When there is no sliding occurring, the friction force can have any value from zero up to F_{max}. Any force smaller than F_{max} attempting to slide one surface over the other is opposed by a frictional force of equal magnitude and opposite direction. Any force larger than F_{max} overcomes the force of static friction and causes sliding to occur. The instant sliding occurs, static friction is no longer applicable and kinetic friction becomes applicable.

An example of static friction is the force that prevents a car wheel from slipping as it rolls on the ground. Even though the wheel is in motion, the patch of the tire in contact with the ground is stationary relative to the ground, so it is static rather than kinetic friction. The maximum value of static friction, when motion is impending, is sometimes referred to as **limiting friction**, although this term is not used universally.

2.4. Kinetic friction

Kinetic (or dynamic) friction occurs when two objects are moving relative to each other and rub together (like a sled on the ground). The coefficient of kinetic friction is typically denoted as μ_k, and is usually less than the coefficient of static friction for the same materials.

Topic Objective:

At the end of this topic student would be able to:

- Define the term internal forces
- Describe the internal forces application in detail

Definition/Overview:

Internal Forces: The internal forces in a section of a body are those forces which hold together two parts of a given body separated by the section. Both parts of the body remain in equilibrium. It follows that internal forces which exist at a section are equivalent to all external forces acting on the particular part of the body.

Key Points:

1. Internal forces

As one can calculate the forces and moments transmitted through joints between members, one can also calculate the internal forces which one part of a member exerts on another. To calculate these internal forces, simply draw a free-body diagram of only part of the member, cutting through the member at the point you are interested in knowing the forces and moments. For example, consider the following member

If you are interested in knowing the forces and moments that are transmitted through the member at point *D*, you can draw the free-body-diagram of the portion to the left of *D* to get

For the body to be in equilibrium one must have

In this example, is the axial force exerted by the right side of the bar on the left side of the bar at *D*, is the shear load exerted by the right side of the bar on the left side of the bar at *D*, is the bending moment exerted by the right side of the bar on the left side of the bar at *D*. The example shows the basic elements of how one find the internal forces at a given point in a member. Like any other constraint, one must introduce a force or a moment for every way in which the motion of one side of the point is restricted by the other side. For example, in the above the right side of *D* exerted restricts the left side from freely moving along the axial direction, transverse to the axial direction, and restricts its free rotation. Consequently, two forces and one moment are introduced to enforce the restriction. The forces and moment applied by the left-hand side onto the right-hand side are equal in magnitude but opposite in direction to the forces applied by the right-hand side on the left-hand side.

Example/Case Study:

Internal forces in a body

The internal forces in a section of a body are those forces which hold together two parts of a given body separated by the section. Both parts of the body remain in equilibrium. It follows that internal forces which exist at a section are equivalent to all external forces acting on the particular part of the body.

All internal forces in the section are usually replaced by a force-couple system , in the centroid C of the cut K.. The force consists of the *axial force* (its line of action is perpendicular to the plane K and *shearing force* lying in the plane K. Accordingly, couple consists of two components the first of which is referred to as the *torque* (its line of action is perpendicular to the plane K) and the second is called the *bending moment* lying in the plane K.

Now, we will restrict our attention to the case in which a body is loaded in just one plane. Moreover we will analyze the internal forces in a very common engineering structure which is referred to as a *beam*. Beams are usually long straight slender prismatic members designed to support transversal loads. The loads may be either concentrated at specific points, or distributed along the entire length or a portion of the beam. We will limit our analysis to beams which are statically determinate supported. The aim of an analysis is to obtain shear V and bending moment M in all cuts K of the beam.
First we determine the reactions at the supports of the beam. Than we cut the beam at K and use the free-body diagram of one of the two parts of the beam. We adopt the sign convention according to Fig. 3. The result of our analysis should be a shear diagram and bending moment diagram representing the shear and the bending moment at any section of the beam. For doing so we use so called Schwedler theorem saying

Where,

w is the distributed load per unit length assumed positive if directed downwards

V is the shear

M is the bending moment

x is the coordinate of the cut oriented from left to right.

We note that the cuts of the beam where the bending moment is maximum or minimum are also the cuts where the shear is zero.

Sample problem: The beam of the lenght $l = 0.7$ m is shown in Fig. 4. It is loaded by the force $F = 400$ N and by partly non-uniformly distributed load characterized by $w_1 = 50\ Nm^{-1}$ and $w_2 = 400\ Nm^{-1}$. The angle = 45°. Determine inner forces at section A-A.

Solution

First we determine the force *S* in the rope. The free-body diagram is shown in Fig. 4. The distributed loads are substituted by forces

They act in the centroids of the respective areas.

The moment equilibrium equation with respect to B yields

and the result is S = 242.67 N.

Second we use the free-body diagram of the right-hand part of the beam for determining the internal forces N, V, M_o.

We have

$$W^x_1 = \quad l\, w_1, \qquad W^x_2 = \qquad l \quad w_2$$

and - for the section A-A -

$$N = S\cos \quad = 171.56 \text{ N}$$

$$V = S\sin \quad - W^x_1 - W^x_2 = 148.23 \text{ N}$$

$$M_o = \quad S\sin \quad - \quad W^x_1 - \quad l\, W^x_2 = 56.76 \; Nm$$

Third we construct shear and moment diagrams according to definitions. The maximum value of

the bending moment M_{omax} = 34.83 Nm occurs in the section where V = 0, namely where x = = 0.2 m from the left side.

Exercise 3.10.2 Internal forces in a beam The simply supported beam has the length l = 0.6 m, a = 0.2 m. It is loaded by the force F = 200 N, by the torque M = 20 Nm, and by uniformly distributed load w = 100 Nm^{-1}.

Determine the shear and moment equations for the beam. Draw shear and moment diagrams. Indicate the section where the bending moment reaches its maximum value.

Solution

x = 0.2 m, M_{omax} = 34.83 Nm.

Exercise 3.10.3 Internal forces in a beam The beam shown is loaded by the forces F_1 = 400 N, F_2 = 500 N, by the torque M = 90 Nm, and by the distributed load w = 5000 Nm^{-1}. Further we know a = 0.3 m, = 30^o. Draw shear and moment diagrams. Indicate the section where the bending moment reaches its maximum value and compute it.

Solution

x = 0.193 m, M_{omax} = 93.44 Nm^{-1}

Exercise 3.10.4 Internal forces in a beam The beam shown is loaded by the torque M = 60 Nm, by uniform distributed load w_1 = 400 Nm^{-1}, and by linearly distributed load w_2 = 1066 Nm^{-1}. The length l = 0.3 m. Draw shear and moment diagrams. Indicate the section where the bending moment reaches its maximum value and compute it.

Solution

x = 0.3 m, M_{omax} = - 48 Nm

Exercise 3.10.5 Internal forces in a beam The beam shown is loaded by linearly distributed load w_0 = 1000 Nm^{-1}. The length a = 0.1 m. Draw shear and moment diagrams. Indicate the section where the bending moment reaches its maximum value and compute it.

Solution

x = 0.43 m, M_{omax} = 23.59 *Nm*

Exercise 3.10.6 Internal forces in a beam The simply supported beam is loaded by sinus-shape distributed load. It is known that l = 0.7 m, w_0 = 800 Nm^{-1}. Draw shear and moment diagrams. Indicate the section where the bending moment reaches its maximum value and compute it.

Solution

x = 0.35 m, M_{omax} = 39.72 *Nm*

Topic Objective:

At the end of this topic student would be able to:

- Define the term potential energy
- Describe the existence of potential energy
- Highlight the gravitational potential energy

Definition/Overview:

Potential Energy: Potential energy can be thought of as energy stored within a physical system. It is called potential energy because it has the potential to be converted into other forms of energy, such as kinetic energy, and to do work in the process. The standard (SI) unit of measure for potential energy is the joule, the same as for work or energy in general.

Key Points:

1. Potential Energy

Potential energy is energy that is stored within a system. It exists when there is a force that tends to pull an object back towards some original position when the object is displaced. This force is often called a restoring force. For example, when a spring is stretched to the left, it exerts a force to the right so as to return to its original, unstretched position. Similarly, when a weight is lifted up, the force of gravity will try to bring it back down to its original position. The initial steps of stretching the spring or lifting the weight both require energy to perform. According to the principle of conservation of energy, energy cannot be created or destroyed; hence this energy cannot disappear. Instead, it is stored as potential energy. If the spring is released or the weight is dropped, this stored energy will be converted into kinetic energy by the restoring force elasticity in the case of the spring, and gravity in the case of the weight.

The more formal definition is that potential energy of a system is the energy of position, that is, the energy a system is considered to have due to the positions of its components in space. For given positions of all other objects of the system, the potential energy is a function of the position of a given object. There are various types of potential energy, each associated with a particular type of force. More specifically, every conservative force gives rise to potential energy. For example, the work of elastic force is called elastic potential energy; work of gravitational force is called gravitational potential energy, work of the Coulomb force is called electric potential energy; work of strong nuclear force or weak nuclear force acting on the baryon charge is called nuclear potential energy; work of intermolecular forces is called intermolecular potential energy. Chemical potential energy, such as the energy stored in fossil fuels, is the work of the Coulomb force during rearrangement of mutual positions of electrons and nuclei in atoms and molecules. Thermal energy usually has two components: the kinetic energy of random motion of particles and potential energy of their mutual positions.

As a general rule, the work done by a conservative force *F* will be

Where, ΔU is the change in the potential energy associated with that particular force. The most common notations for potential energy are *PE* and *U*. Electric potential (commonly denoted with a *V* for voltage) is the electric potential energy per unit charge.

1. Reference level

The potential energy is a function of the state a system is in, defined up to a constant term. This term can be chosen such that the formulas are easiest, and/or such that for a particular state the potential energy is zero. Typically the term is chosen such that the potential energy depends on the *relative* positions of its components only. In the case of inverse-square-law forces, a common choice is to let the potential energy approach zero when the distances between all bodies tend to infinity.

2. Gravitational potential energy

Gravitational energy is the potential energy associated with gravitational force. If an object falls from point A to point B inside a gravitational field, the force of gravity will do positive work on the object and the gravitational potential energy will decrease by the same amount.

For example, consider a book, placed on top of a table. When the book is raised from the floor to the table, the gravitational force does negative work. If the book is returned back to the floor, the exact same (but positive) work will be done by the gravitational force. Thus, if the book is knocked off the table, this work (called potential energy) goes to accelerate the book (and is converted into kinetic energy). When the book hits the floor this kinetic energy is converted into heat and sound by the impact.

The factors that affect an object's gravitational potential energy are its height relative to some reference point, its mass, and the strength of the gravitational field it is in. Thus, a book lying on a table has less gravitational potential energy than the same book on top of a taller cupboard, and less gravitational potential energy than a heavier book lying on the same table. An object at a certain height above the Moon's surface has less gravitational potential energy than at the same height on Earth because the Moon's gravity is weaker. (This follows from Newton's law of gravitation because the mass of the moon is much smaller than that of the Earth.) It is important to note that "height" in the common sense of the term cannot be used for gravitational potential energy calculations when gravity is not assumed to be a constant. The following sections provide more detail.

The strength of a gravitational field varies with location. However, within a small range of distances from the center of the source of the gravitational field, this variation in field strength is negligible and we can assume that the force of gravity on a particular object is constant. Near the surface of the Earth, for example, we assume that the acceleration of gravity is a constant $g = 9.8$ m/s^2. If we assume that the force of gravity is constant, a simple expression for gravitational potential energy can be derived using the $W = Fd$ equation for work, and the equation

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When accounting only for mass, gravity, and altitude, the equation is:

Where:

m is the mass of the gravitated object, in kilograms

g is standard gravity, in m / s^2, and

h is the altitude of the gravitated object, in metres

Hence, the potential difference is

.

However, if the force of gravity varies too much for this approximation to be valid, then we have to use the general, integral definition of work in order to determine gravitational potential energy. The reference point where $U = 0$ is set at an infinite distance away from the source of the gravitational field provided by mass m_2. The gravitational potential energy of a system of masses m_1 and m_2 at a distance *R* is

Although this would not be an inertial frame of reference, for the computation of the potential energy we can keep the position of one mass fixed and find the equation by integrating the gravitational force (whose magnitude is given by Newton's law of gravitation) with respect to the distance of the object *r* from the gravitating body from r = R to .
The total potential energy of a system of *n* bodies is found by adding for all $n (n - 1) / 2$ pairs of two bodies the potential energy of the system of those two bodies.

Considering the system of bodies as the combined set of small particles the bodies consist of, and applying the previous on the particle level we get the negative gravitational binding energy. This potential energy is more strongly negative than the total potential energy of the system of bodies as such since it also includes the negative gravitational binding energy of each body. The potential energy of the system of bodies as such is the negative of the energy needed to separate the bodies from each other to infinity, while the gravitational binding energy is the energy needed to separate all particles from each other to infinity.