

Lecture 1 Game Plan

- Introduce the course
- Logistics / expectations
- More examples

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What are Games?

*"No Man is an Island,
Entire of Itself"*

- John Donne, 1624

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What are Games?

"In War the Will Is Directed at an Animate Object That Reacts."

- Karl Von Clausewitz in "On War", 1832

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What is Game Theory?

- Study of rational behavior in situations in which your choices affect others & their choices affect you (so-called “games”)
- Bad news:
Knowing game theory does not guarantee winning
- Good news:
Framework for thinking about strategic interaction

Games We Play

- Driving coordination
- Penalty Kicks hunter & hunted
- GPA trap prisoner's dilemma
- Doing the dishes war of attrition
- Mean professors commitment
- Group projects free-riding
- Dating hidden information

Games Businesses Play

- Standards adoption coordination
- Audits hunter & hunted
- Price wars prisoner's dilemma
- Standards wars war of attrition
- Capacity expansion commitment
- Pollution abatement free-riding
- External financing hidden information
- FCC spectrum auctions

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Why Study Game Theory?

Because the press tells us to...

"As for the firms that want to get their hands on a sliver of the airwaves, their best bet is to go out first and hire themselves a good game theorist."

The Economist, July 23, 1994 p. 70

"Game Theory, long an intellectual pastime, came into its own as a business tool."

Forbes, July 3, 1995, p. 62.

"Game theory is hot."

The Wall Street Journal, 13 February 1995, p. A14

Why Study Game Theory?

Because consultants tell us to:

- John Stuckey & David White – McKinsey Sydney
- Tom Copeland – Monitor

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Why Study Game Theory?

Because business leaders tell us to:

- Raymond W. Smith (Bell Atlantic Chairman during 1990s)

"At Bell Atlantic, we've found that the lessons of game theory give us a wider view of our business situation and provide us a more nimble approach to corporate planning. We call this system, quite simply, the 'manage the business' process."

-- in *"Business as war game: a report from the battlefield"*, Fortune, Sep. 1996

Why Study Game Theory?

- Because we can formulate effective strategy...
- Because we can predict the outcome of strategic situations...
- Because we can select or design the best game for us to be playing...

Game Theory

Administrivia

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Course Information

- Materials on Sloan class server
- Instructor: David McAdams

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Prerequisite: 15.010

- In particular, I assume that you are very comfortable
 - representing games with payoff matrix and/or decision tree
 - computing reaction curves
 - computing equilibrium given payoff matrix and/or decision tree

Grading

- Problem Set 15%
- Strategy Memo 15%
- Real-World Application 25%
- Game Participation 20%
- Final Exam 25%

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Team Assignments

Target 3-4 students per team

1. *Problem Set*
2. *Strategy Memo*
3. *Real-World Application*

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Group Assignments

1. *Problem Set:* Qwest Bond Swap

“Design a debt tender offer to minimize the risk of bankruptcy and maximize shareholder equity.”

Group Assignments

2. *Strategy Memo: Airline Strategy*

"American Airlines needs you to assess a plan to extend 'More Room Throughout Coach' to the older and smaller Super 80 fleet."

Group Assignments

3. *Real-World Application*

“From your analysis, develop strategy advice for one of the players in the game or for a party interested in the outcome.”

- a. *Mixed Strategies.*
- b. *Sequential / Repeated Games.*
- c. *Strategic Moves.*
- d. *Information.*

Online Games

- Six online games, prior to:
 - Lectures 2,3,4,5,9,10
- Participate by midnight before lecture
- No preparation: 5-15 minutes to play
- Results revealed in class
- Graded for participation only

In-Class Games

- Four in-class games
 - Lectures 2,3,6,8

- Require preparation outside of class
 - 1-2 page worksheet for you to complete and hand in
 - graded for participation only

Final Exam

- Take-home exam
 - Open notes / Open book
 - Due at beginning of last class
 - Practice exam will be provided

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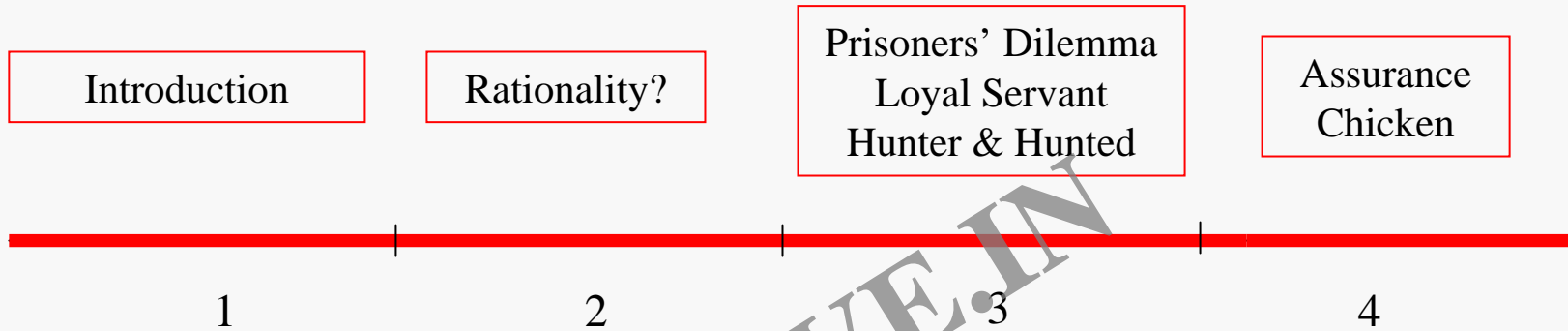
Wait List & Auditing

- TA will be managing the wait list
- As a courtesy, please inform the TA
 - if you want off the wait list
 - if you think there's 50% or more chance you will drop
- No auditors please

Outline of Course

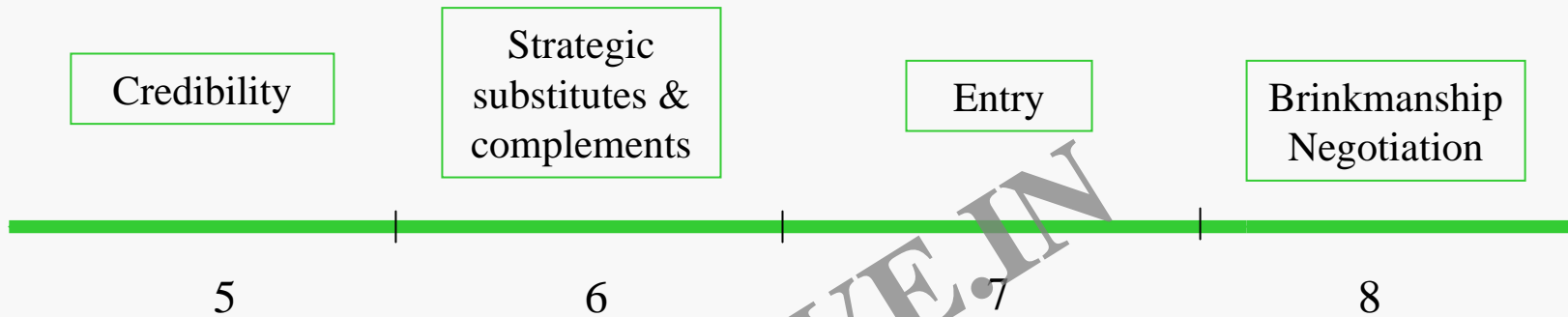


Part I: Foundations



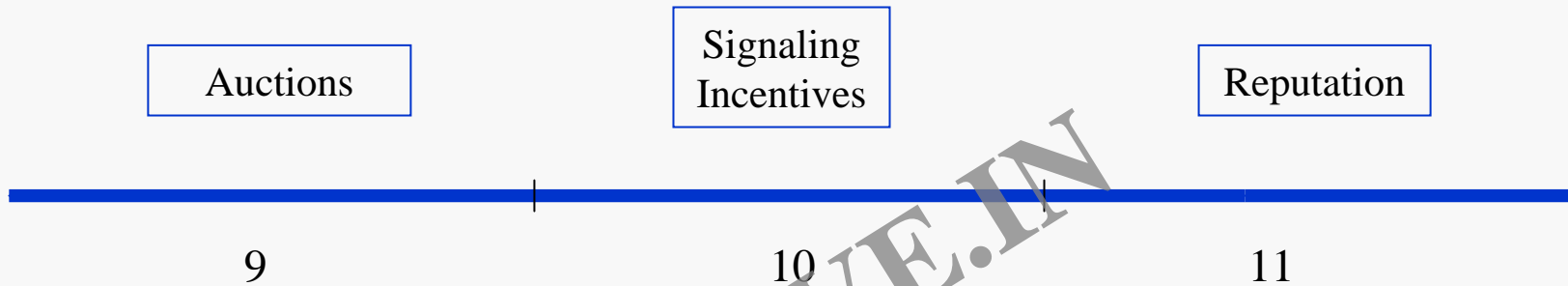
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Part II: Commitment



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Part III: Information



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Game Theory = Interactive Decision Theory

■ Decisions

- You take the world as given and make the best decision for yourself

■ Games

- Your best decision depends on what others do, and what they do may depend on what they think you do ...

Overview

1. Key elements of a game
2. Anticipating others' behavior
3. Acquiring advantage

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Key Elements of a Game

- **Players:** Who is interacting?
- **Strategies:** What are their options?
- **Payoffs:** What are their incentives?
- **Information:** What do they know?
- **Rationality:** How do they think?

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How to anticipate others' behavior in a game

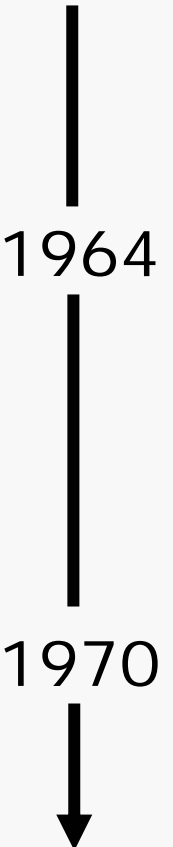
- **Evolution:** If non-strategic and adaptive, play repeatedly (or observe past play)
- **Dominance:** If never play a strategy that is always worse than another
- **Rationalizability:** If play optimal given some beliefs about what others play (and what others believe)
- **Equilibrium:** If play optimal given *correct* beliefs about others

Themes for acquiring advantage in games

1. **Commitment / strategic moves:**
Credibility, threats, and promises
2. **Leveraging limited rationality:**
Reputation and surprise
3. **Exploiting incomplete information:**
Signaling, selection, and info cascades

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Cigarette Advertising on TV

- 
- A vertical timeline on the left side of the slide. It starts with a vertical line at the top, followed by a downward-pointing arrow. The year '1964' is written to the left of the arrow, and '1970' is written further down. The arrow ends in a downward-pointing arrowhead.
- All US tobacco companies advertised heavily on television
 - 1964 Surgeon General issues official warning
 - Cigarette smoking may be hazardous
 - Cigarette companies' reaction
 - Fear of potential liability lawsuits
 - 1970 Companies strike agreement
 - Carry the warning label and cease TV advertising in exchange for immunity from federal lawsuits.

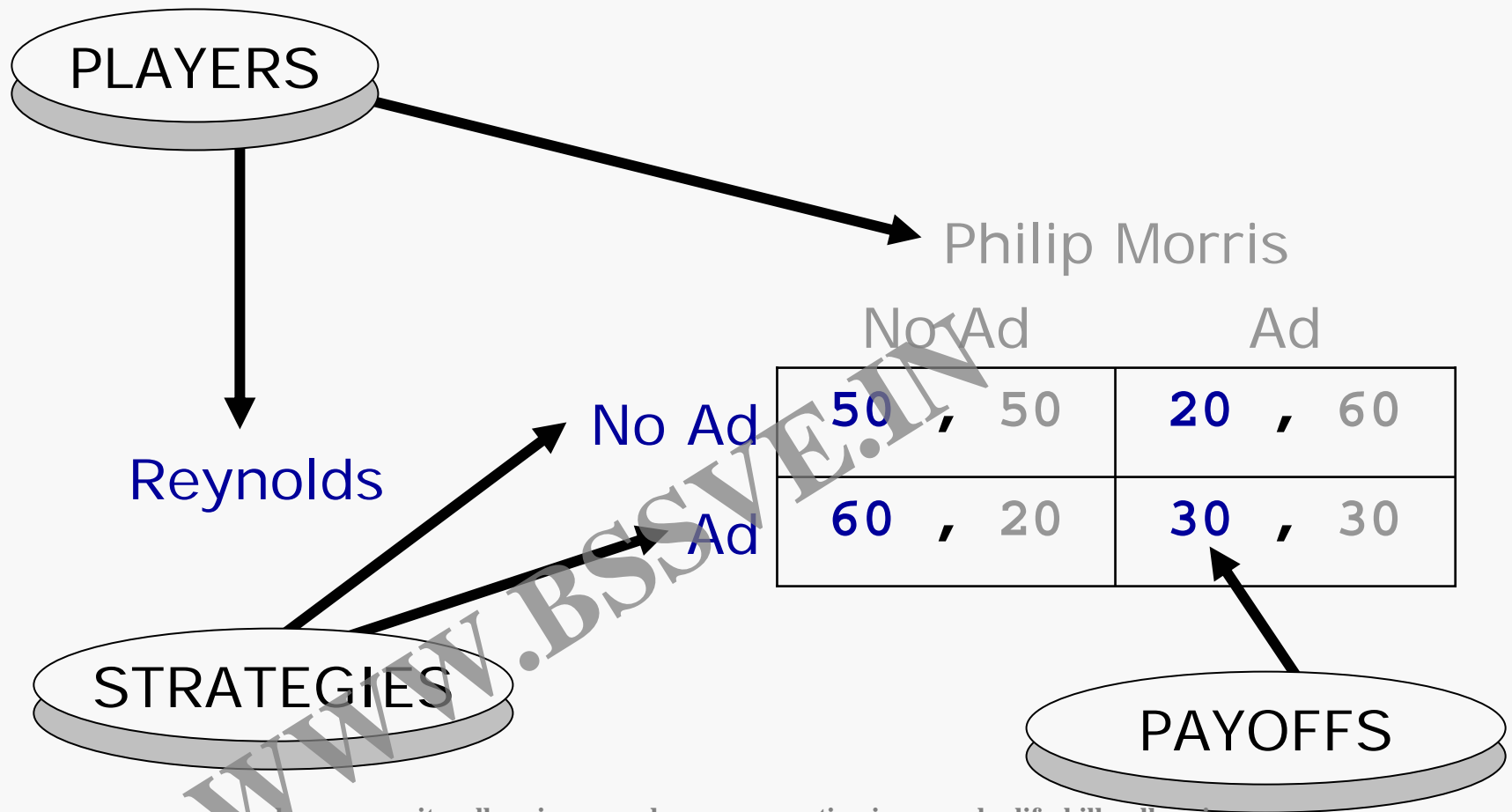
Strategic Interactions

- Players: Reynolds and Philip Morris
- Strategies: { Advertise , Do Not Advertise }
- Payoffs: Companies' Profits

- Each firm earns \$50 million from its customers
- Advertising costs a firm \$20 million
- Advertising captures \$30 million from competitor

- How to represent this game?

Payoff Table



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Best responses

		Philip Morris	
		No Ad	Ad
Reynolds	No Ad	50 , 50	20 , 60
	Ad	60 , 20	30 , 30

- Best response for Reynolds:
 - If Philip Morris advertises: advertise
 - If Philip Morris does not advertise: advertise
- Advertise is dominant strategy!
- This is another Prisoners' Dilemma

What Happened?

- After the 1970 agreement, cigarette advertising decreased by \$63 million
- Profits rose by \$91 million
- Why/how were the firms able to escape from the Prisoner's Dilemma?

Changing the Game thru Gov't-Enforced Collusion?

		Philip Morris	
		No Ad	Ad
Reynolds	No Ad	50 , 50	20 , 60
	Ad	60 , 20	30 , 30

- The agreement with the government *forced* the firms not to advertise.
- The preferred outcome (No Ad, No Ad) then was all that remained feasible

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Example: MBA Recruitment

- The message at Sloan orientation: “start looking immediately for a job”
 - It could be worse: in the 1920’s, people admitted to Law School were being offered clerkships *before ever setting foot on campus*
- Why does recruitment start so early?

Vicious Cycle of Adverse Selection

- Early-moving firms will hire students who tend to be above-average
- The remaining population of students is on average worse than before, which increases the incentive to move early!
- Why doesn't Sloan administration act as third-party enforcer to allow us to escape this Prisoners' Dilemma?

Third-Party Enforcement?

- Firms and students have no incentive to report an early hire, if this will be punished
 - No way for Sloan to be effective enforcer since it can't identify deviants
- Does this mean that “unraveling” is inevitable in the MBA recruitment market?

Jump the Gun or Not?

- Downside of moving early: firm gets less precise information about student quality
- Upside of moving early: more students are available to be hired
- Jumping the gun is not a dominant strategy if downside outweighs upside. For example:
 - students can provide strong signals while in school
 - students outnumber job openings

Strategies for Studying Games

- Two general approaches
 - Case-based
 - Pro: Relevance, connection of theory to application
 - Con: Generality
 - Theory
 - Pro: General principle is clear
 - Con: Applying it may not be

Approach of this course

- Between theory and cases
- Lectures organized around general principles
- Illustrated with cases and game-playing
 - Cases are “stripped down” to essentials
- Frequent reality checks
 - When does theory work?
 - When doesn't it?
 - Why doesn't it?

The Uses of Game Theory

- Explanatory
 - A lens through which to view and learn from past strategic interactions
- Predictive
 - With many caveats
- Prescriptive
 - The main thing you'll take out of the course is an ability to think strategically

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Online Game #1

- Play Online Game #1 prior to midnight before next lecture.
- Follow instructions on handout
 - Sign up / create password on game site
 - Then play the game (no preparation)

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In-Class Game Next Time

- Prepare for “Urn Game” to be played in class next lecture.
 - See handout

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Lecture 2 Game Plan

- Question the fundamental assumptions of game theoretic analysis
 1. Rational decision-making
 2. Common knowledge of rationality
 3. Nash equilibrium
- Begin the rebuilding process
 - *Dominant strategies* when 1,2,3 may fail

Rationality?

"Only the Paranoid Survive"

- Andy Grove, Intel Co-founder

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Rationality?

- Most economic analysis assumes “rationality” of decision-makers, i.e. that you make decisions by
 1. forming a belief about the world
 2. choosing an action that maximizes your welfare given that belief
- In principle, requires enormous powers of imagination and computation.

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And Common Knowledge of Rationality??

- Most game-theoretic analysis makes the further assumption that players' rationality is common knowledge
 - Each player is rational
 - Each player *knows* that each player is rational
 - Each player *knows that each player knows* that each player is rational
 - Each player *knows that each player knows that each player knows* that each player is rational
 - Each player *knows that each player knows that each player knows that each player knows* that each player is rational
 - Etc. etc. etc.

And Correct Beliefs?!?

- Nash equilibrium assumes that each player has *correct beliefs about what strategies others will follow*
- Implicitly this is saying that, in novel strategic situations, each player knows what the other believes
- Requires all players to thoroughly understand each other

Online Game #1

The Beauty Contest

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In-Class Game

The Urn Game

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Urn Game: Rules

- Two indistinguishable urns
 - “Urn W” has two white balls, one yellow
 - “Urn Y” has two yellow balls, one white
- TA will flip coin to choose an urn
- You must guess which urn it is after seeing one ball from the urn *and after hearing all guesses of those before you*
- Your goal is to “CHOOSE WISELY”, i.e. to make the correct guess

Urn Game: How We'll Play

- Eight students will be called per round (new urn-draw each round)
 - Please line up in the front of the room
- At your turn, draw out a ball without looking at any others and without showing the ball to anyone else
- Return the ball to the urn, write your guess on provided sheet, then give the sheet to me

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“Groupthink”

- The Urn Game illustrates how conformity can be rational for individuals, even when they don't care what others do per se.
- The decisions made by others convey some information → Rational individuals may ignore their own information
 - “*informational cascade*” when this happens

Conformity

- Observation: People in a group often do (and believe) the same thing as people around them.
- There are several natural reasons for this other than info cascades, including:
 - People may simply prefer doing the same thing (or prefer avoiding being different)
 - People may simply know the best thing for them to do, so everyone does that
- In these cases, we would expect group behavior to be relatively stable over time.

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“Paradigm Shift”

- But in an informational cascade, *rational* individuals' decisions convey relatively little info
 - If the first two people say “Urn W”, everyone else will say “Urn W” regardless of their own ball's color
→ 100 people saying “Urn W” gives same info as 2 people saying “Urn W”
- Better info on a few “irrational” individuals can swing behavior of the entire group

“Path Dependence”

- The behavior of early-movers has a disproportionate effect on the group
- We should therefore expect different groups to make different (possibly wrong) decisions in the same setting

Summary of Info Cascades

- Even when individuals are rational, groups may not be
- *Rationality* of group decision-makers can be an especially strong assumption

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Dominant Strategies

*"I'll make him an offer he
can't refuse."*

- The Godfather

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Recall: Cigarette Ad Game

		Philip Morris	
		No Ad	Ad
Reynolds	No Ad	50 , 50	20 , 60
	Ad	60 , 20	30 , 30

Reynolds' best strategy is Ad regardless of what Philip Morris does → Ad is "*dominant strategy*"

Dominant Strategies and Rationality

- If *you* are rational, you should play your dominant strategy. Period.
- No need to think about whether others are rational, etc.
- Rationality + dominant strategies implies Nash equilibrium
 - no need for common knowledge or correct beliefs

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Warren Buffett's "Billionaire's Buyout Plan"

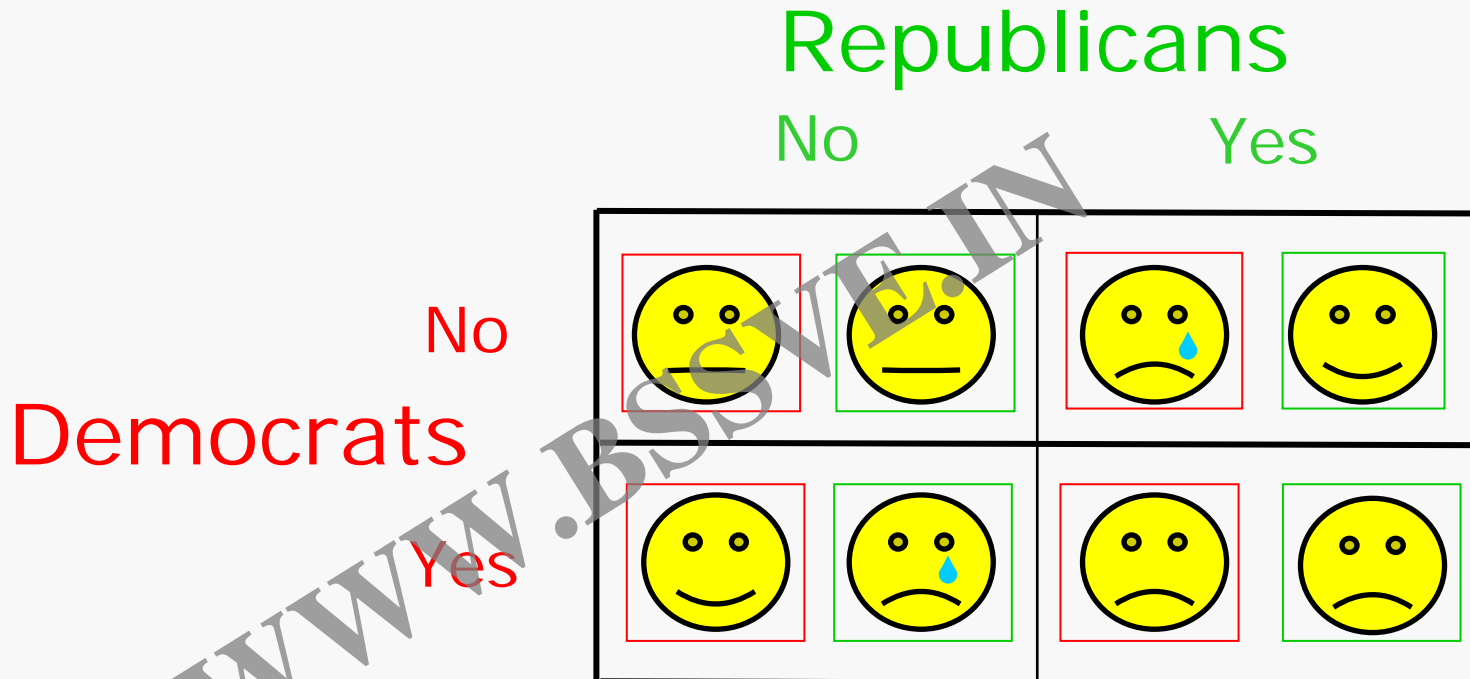
- A campaign finance reform bill banning soft money is about to come to a vote:

"Suppose some eccentric billionaire (not me, not me!) makes the following offer. If the bill is *defeated*, this E.B. will donate \$1 Billion in soft money to the party that delivers the most votes to getting the reform passed"

-- Buffett, Warren. "The Billionaire's Buyout Plan,"
The New York Times, 10 September 2000.

Billionaire's Buyout Plan as Prisoners' Dilemma

For simplicity, suppose that there is one Democrat and one Republican and both need to vote Yes for the reform to pass.



Summary

- Nash equilibrium is not the right concept for some strategic situations
 - Real players make mistakes or, for other reasons, may fail to be “rational”
- Yet dominant strategies give a clear prescription of what to do, regardless.
- *Next time:* Continue rebuilding and applying the notion of Nash equilibrium.

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Online Game #4 (Monitoring Game)

- Play Online Game #4 prior to midnight before next lecture.
- Note: We are *not* playing the games in their numerical order!!

In-Class Game Next Time

- Prepare for “Bluffing Game” to be played in class next lecture.
 - See handout

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Lecture 3 Game Plan

- Nash equilibrium
 - ... in pure strategies & mixed strategies
 - ... how NE play arises from rationality
 - ... how NE play can arise from evolution

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Nash Equilibrium

■ Nash Equilibrium:

- A set of strategies, one for each player, such that each player's strategy is a best response to others' strategies

■ Best Response:

- The strategy that maximizes my payoff given others' strategies.

■ Everybody is playing a best response

- No incentive to unilaterally change my strategy

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Some Prototypical Games

- Prisoners' Dilemma
- price war

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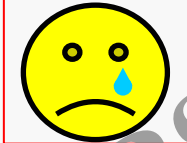



Example: SUV Price Wars

“General Motors Corp. and Ford Motor Co. slapped larger incentives on popular sport-utility vehicles, escalating a **discounting war in the light-truck category** ... Ford added a \$500 rebate on SUVs, boosting cash discounts to \$2,500. The Dearborn, Mich., auto maker followed GM, which earlier in the week began offering \$2,500 rebates on many of its SUVs.”

-- Wall Street Journal, **January 31, 2003**

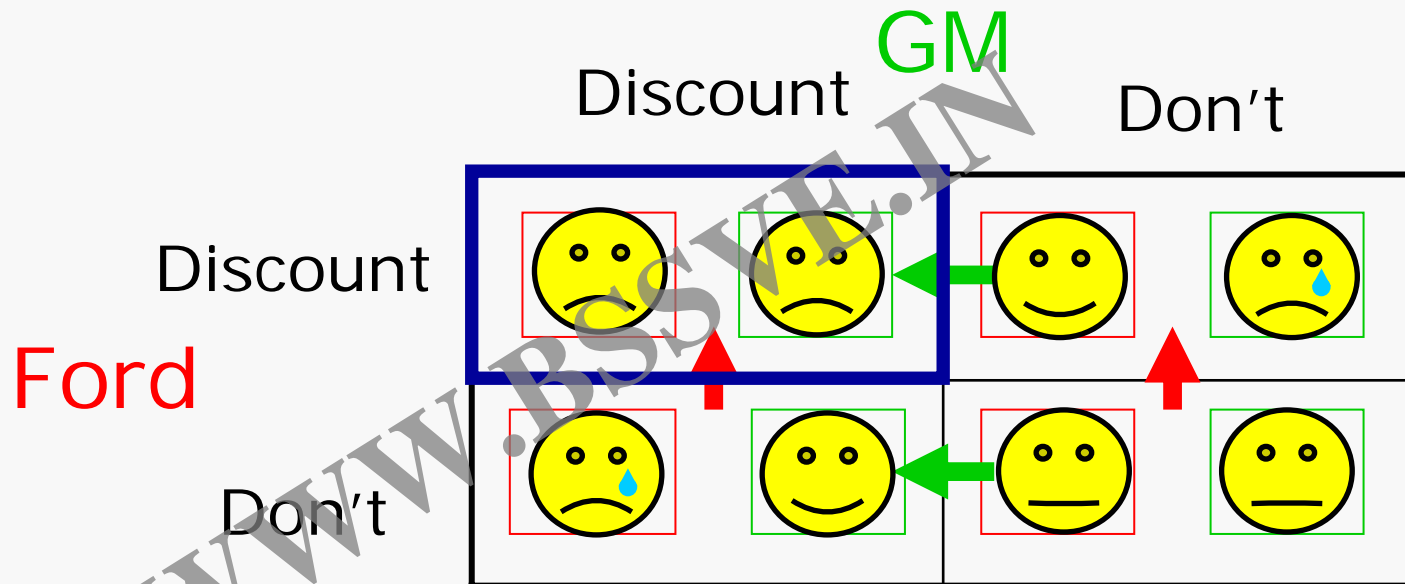
SUV Price Wars: The Game

GM

		Discount	Don't
Ford	Discount	 	 
	Don't	 	 

SUV Price Wars: Outcome

- Each firm has a unilateral incentive to discount but neither achieves a pricing advantage.



“Red Queen Effect”

“It takes all the running you can do to keep in the same place” – Red Queen to Alice

From Carroll, Lewis. *Alice's Adventures in Wonderland*.









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Prisoners' Dilemma

SUV Price War is a “prisoners' dilemma” game:

1. Both firms prefer to Discount regardless of what other does. (Discount is a *dominant strategy*.)
2. BUT both firms are worse off when they both Discount than if they both Don't.

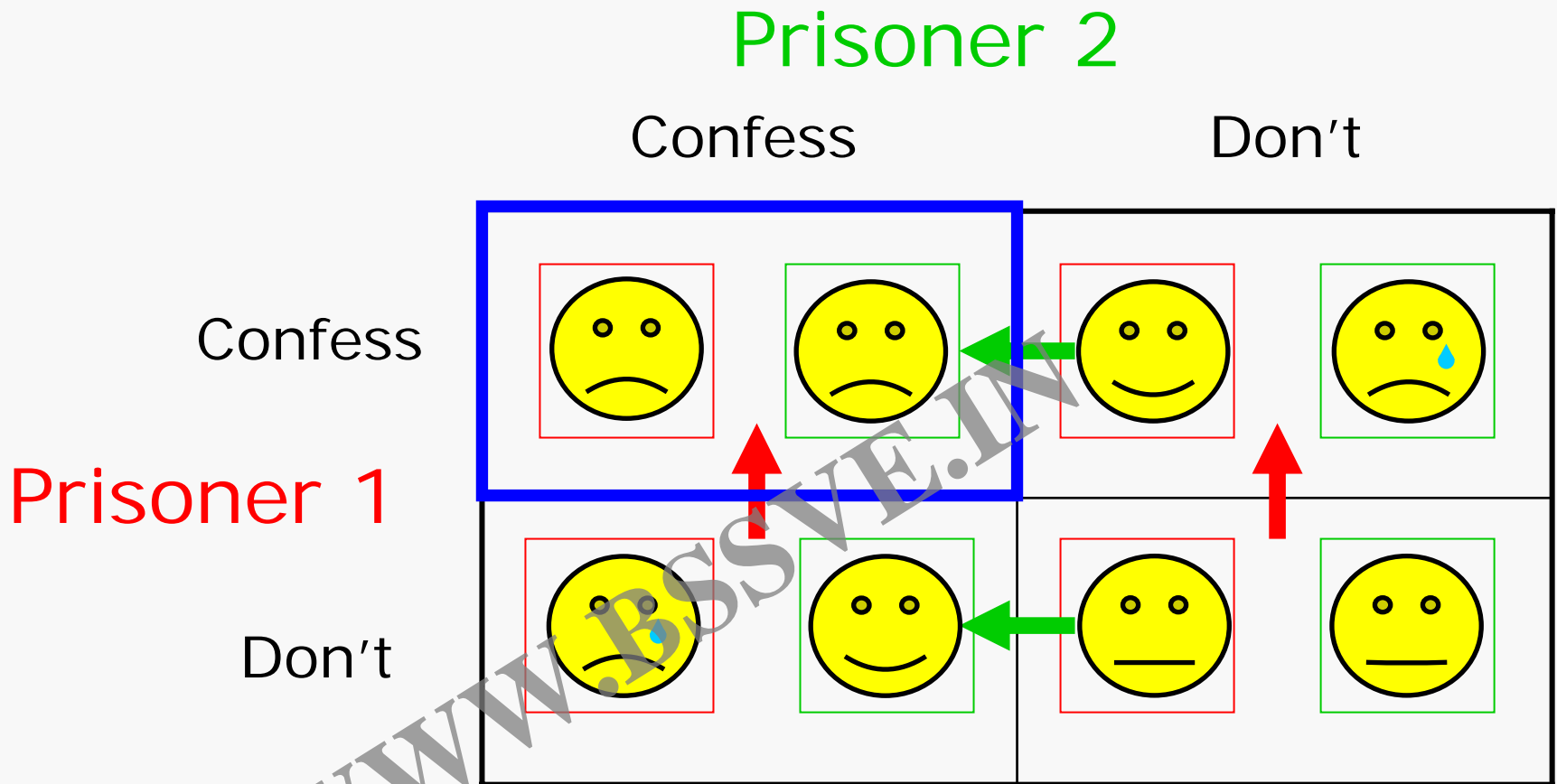
Prisoners' Dilemma Game

		Prisoner 2	
		Confess	Don't
Prisoner 1	Confess	 	 
	Don't	 	 

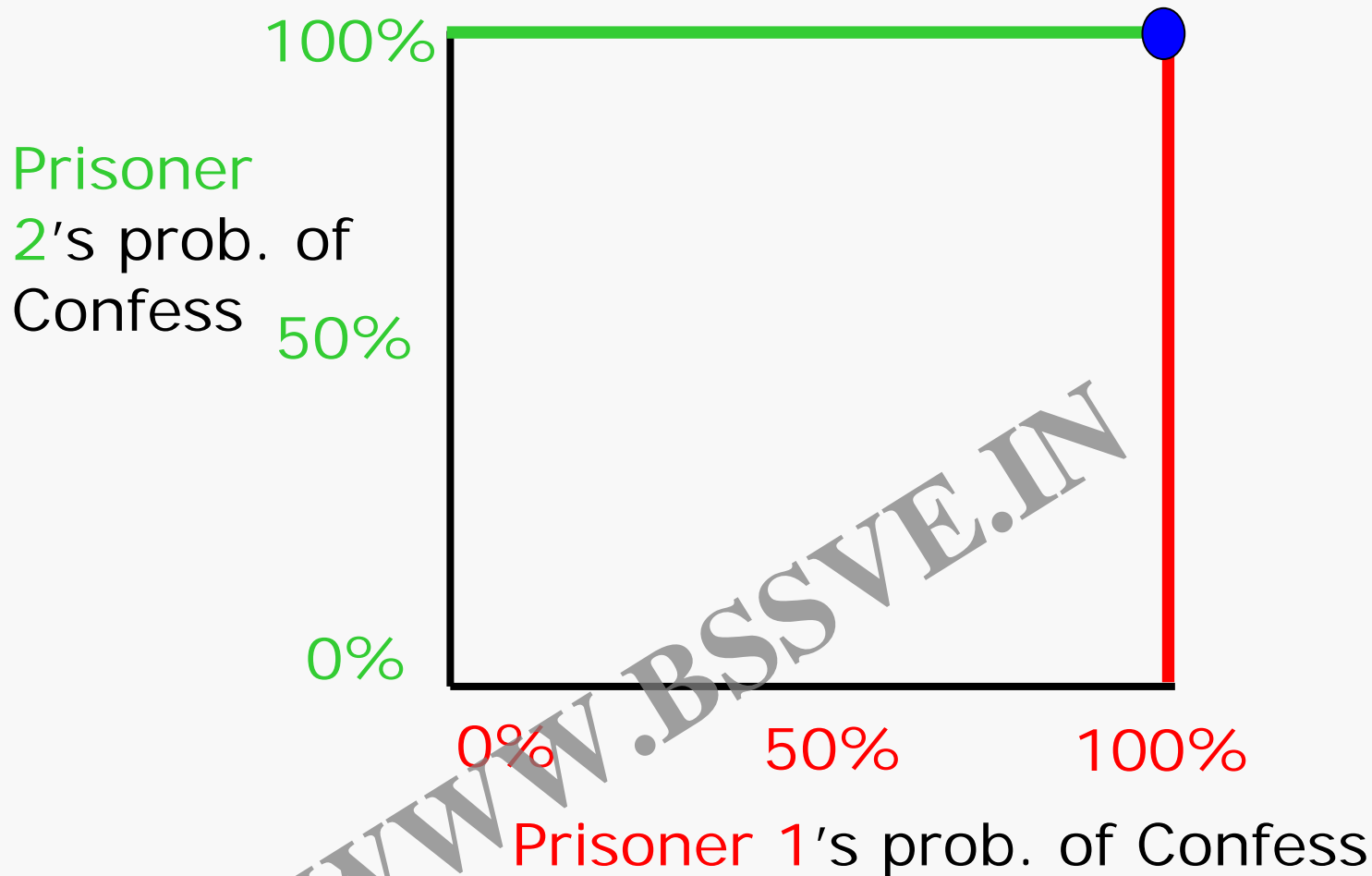
■ Key features:

- Both players have a dominant strategy to Confess
- BUT both players better off if they *both* Don't

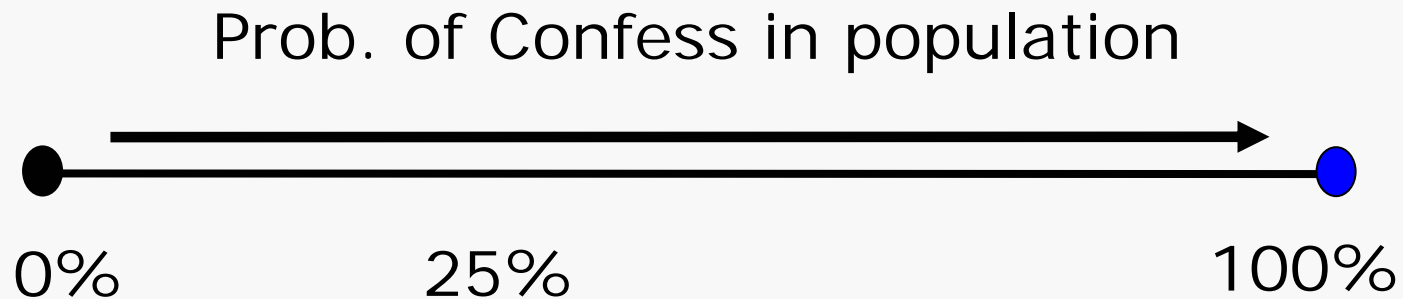
Prisoners' Dilemma Game



Reaction Curves in Prisoners' Dilemma



Evolution in Prisoners' Dilemma (One Population)



- **Row** and **Col** players are drawn from the same population
- Those who Confess get higher payoff, so Confess dominates the population

Some Prototypical Games

- Prisoners' Dilemma
- price war
- **Loyal Servant**
- **defensive innovation**

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Soft & Chewy Cookies

A cookie store is a bad idea. Besides, the market research reports say America likes crispy cookies, not soft and chewy cookies like yours.

-Response to Debbi Fields' idea of starting Mrs. Fields' Cookies, 1977

Soft & Chewy Cookies

- Supermarket cookies tend to be crispy, not chewy
- Duncan Hines (owned by P&G) entered with a chewy cookie [1984]
- How did Nabisco and Keebler respond?

Soft & Chewy Wars

- Nabisco and Keebler rolled out their own soft and chewy varieties:
 - Keebler Soft Batch
 - Nabisco Chips Ahoy! Chewy

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Soft & Chewy Settlement

- Duncan Hines brings patent-infringement suit alleging industrial espionage by Keebler, Nabisco and Frito-Lay [1984]
- Companies agree to pay P&G \$125 million, then the most ever reported to settle a patent lawsuit [1989]

Soft & Chewy Retreat

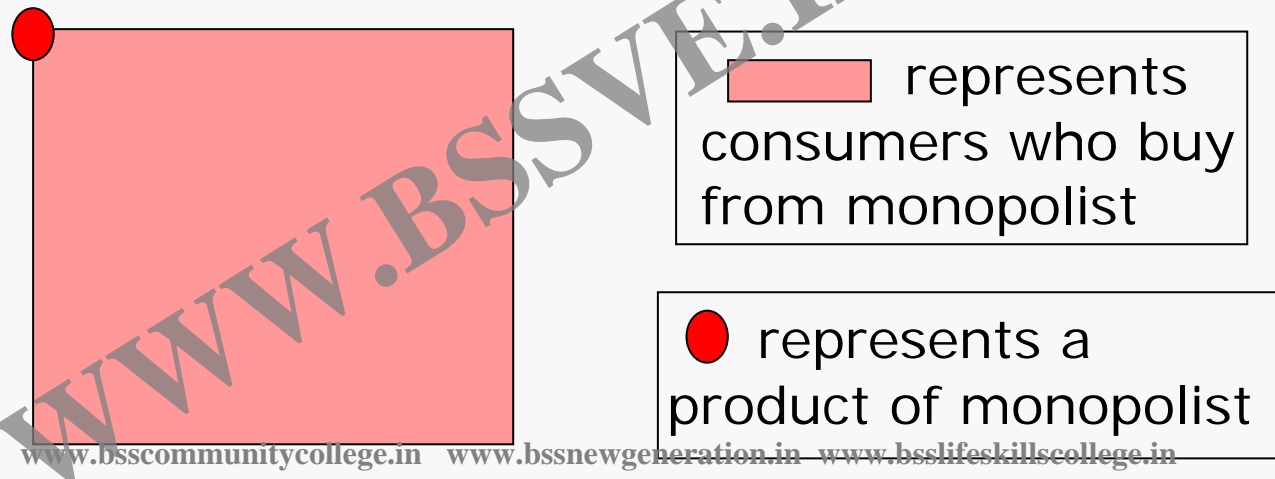
- P&G sells Duncan Hines to Aurora [1997]
“This agreement is a win-win. Consumers will still be able to buy great Duncan Hines products, now through Aurora Foods, and we can focus on the strategic opportunities we've established for our food and beverage business.”

-Steve Donovan, P&G VP, food and beverages.

Source: Larkin, Patrick. "Duncan Hines Sold: Columbus Firm Buys P&G Brand." *The Cincinnati Post*, 8 December 1997, <http://www.cincy.post.com/business/1997/pg120897.html> (accessed July 14, 2004).

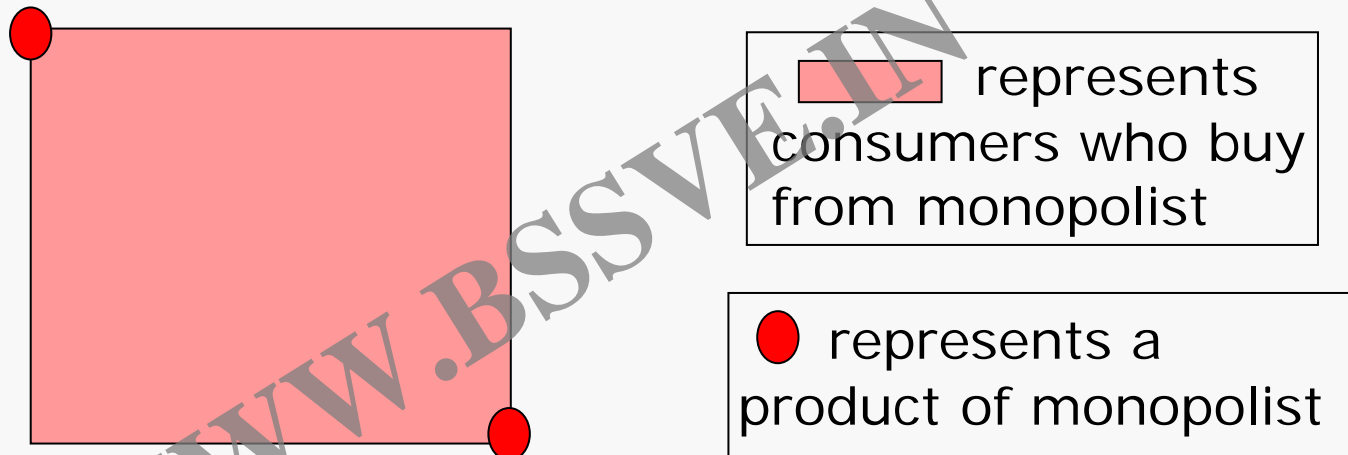
Defensive Innovation

- A monopolist's incentive to innovate increases as it faces innovative entrants.
- Consider case of *product variety*
 - consumers represented as points in a square
 - they buy whichever product is closest



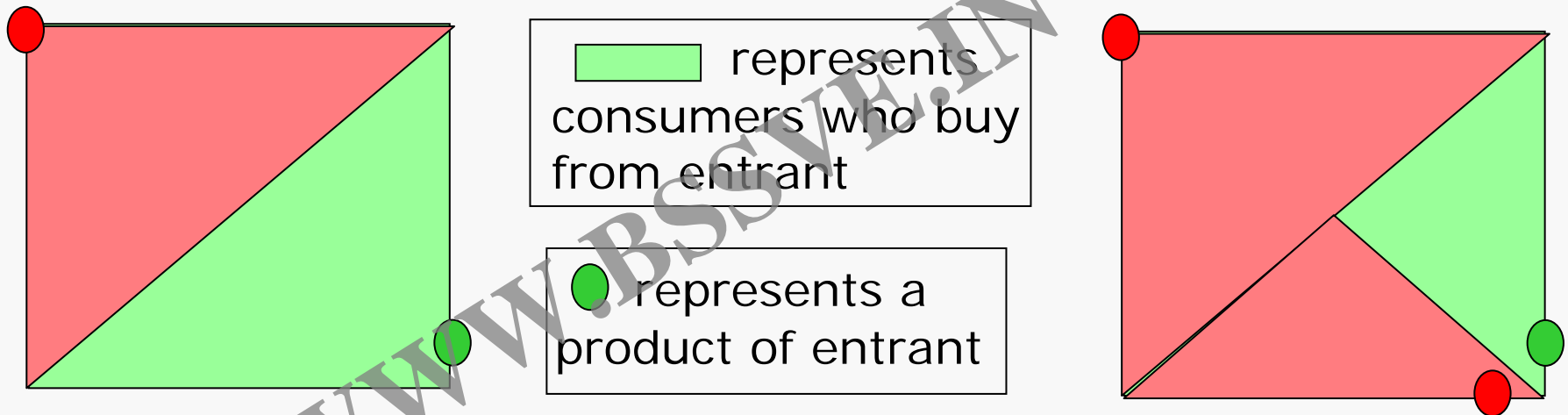
Product Variety Game

- Without any entry, the monopolist doesn't gain from introducing a new product
 - To keep things simple, we suppose price is fixed



Product Variety Game

- Now suppose an entrant comes in with a new variety. Now “landing on the entrant” keeps some customers
 - Incentive to introduce new variety if entry
 - Will the other firm enter anyway?



Defensive Innovation: Summary So Far

Monopolist

Innovate

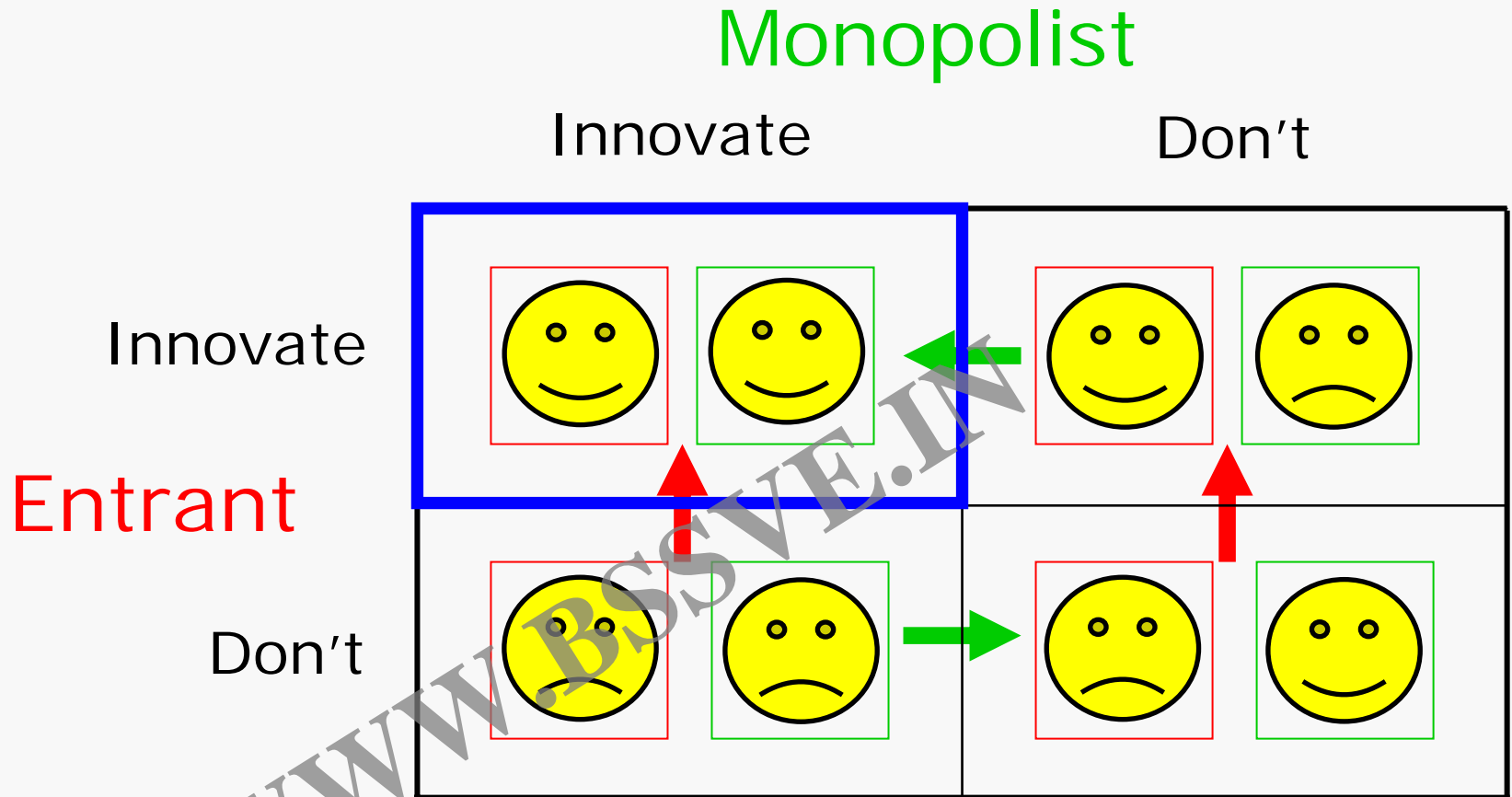
Don't

Innovate
Entrant

Don't

		Innovate		Don't	
Entrant	Innovate	??	😊	😊	😞
	Don't	😞	😞	😞	😊

Case I: Entrant Wants to Innovate Anyway



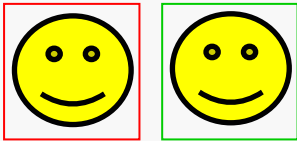
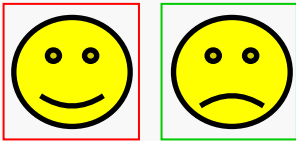
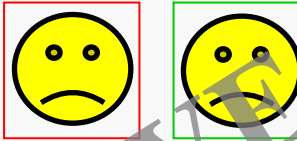

What Can *Conceivably* Happen?

1. ... if both Monopolist and Entrant are *rational* (only)
 - (Innovate, Innovate) or (Innovate, Don't)
 - latter requires monopolist to mistakenly believe that entrant will not innovate

2. ... if rationality is *common knowledge*?
 - (Innovate, Innovate) only since monopolist knows entrant is rational!

Loyal Servant Game*

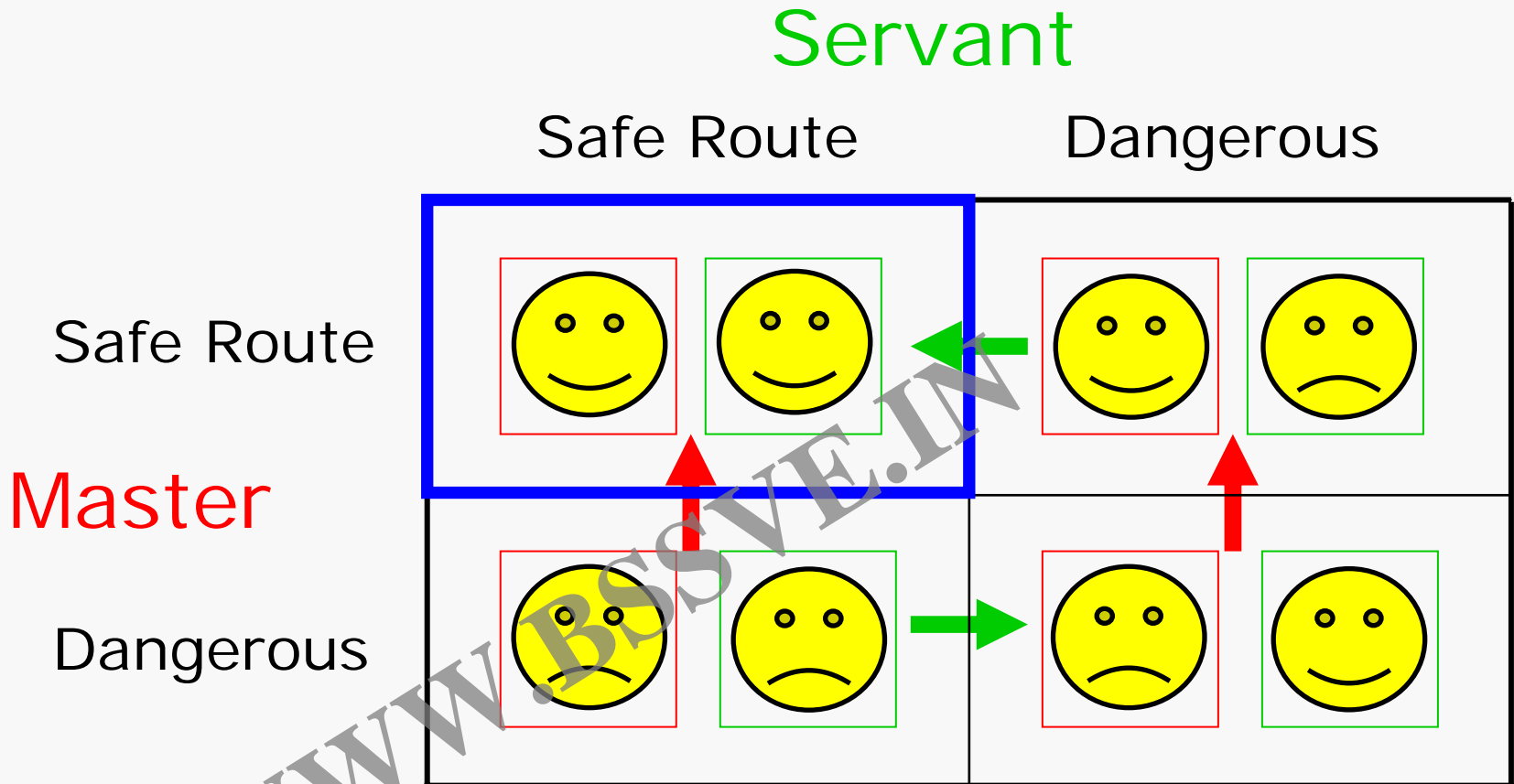
Servant

	Safe Route	Dangerous
Safe Route Master		
Dangerous		

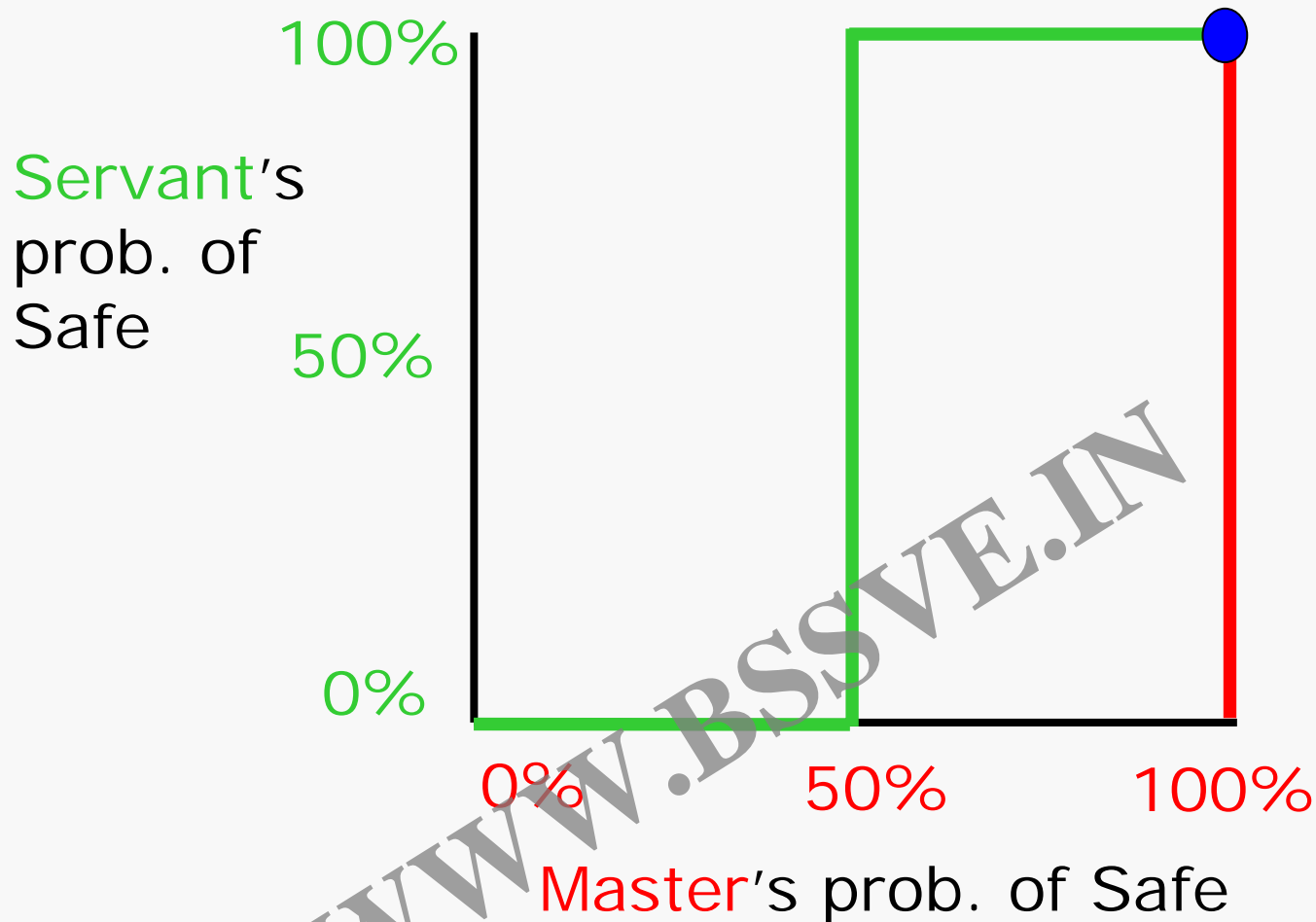
■ Key features:

- One player (Master) has dominant strategy
- Other player (Servant) wants to do the same thing as Master

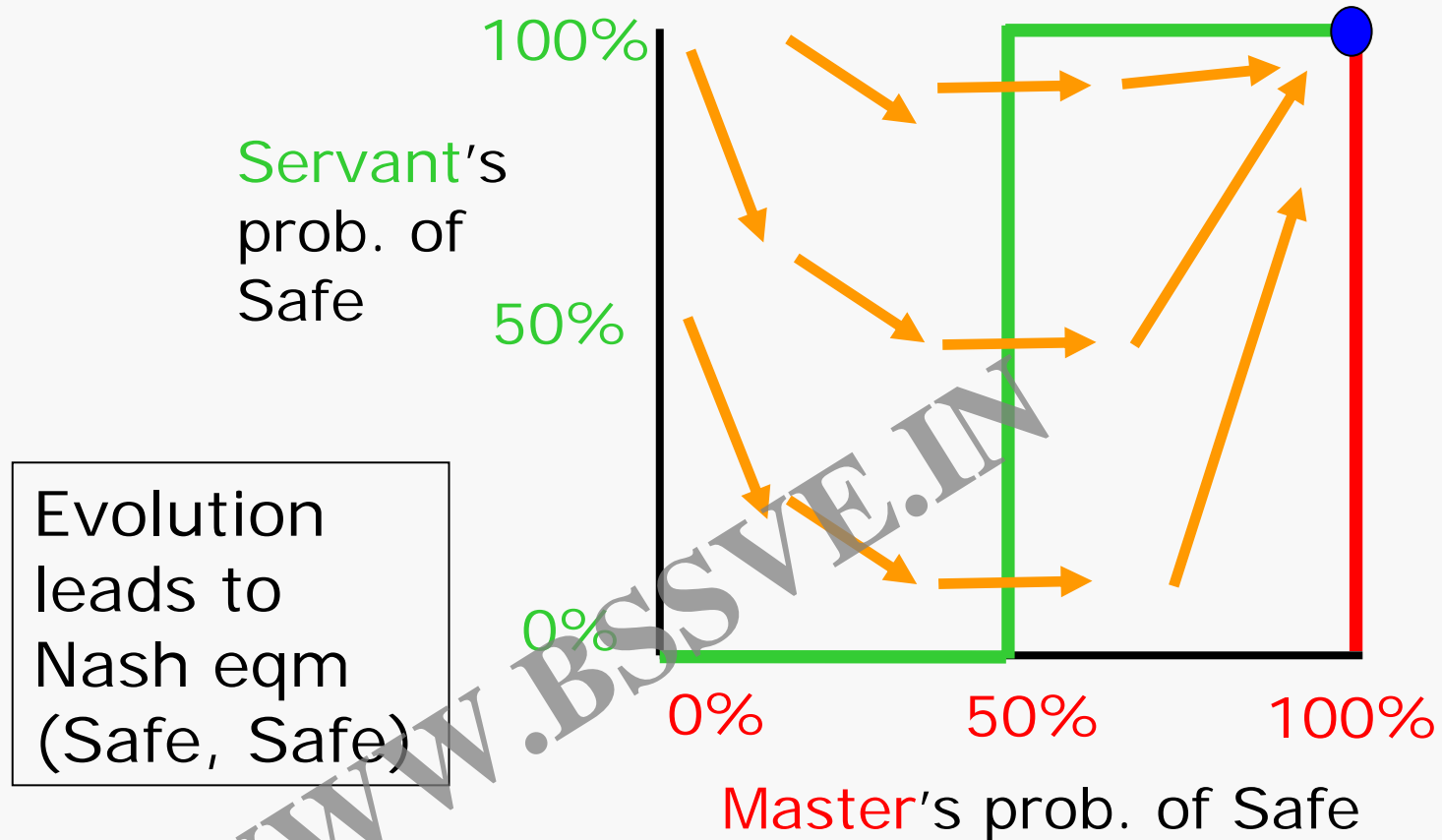
Loyal Servant Game



Reaction Curves in Loyal Servant Game



Evolution in Loyal Servant Game (Two Populations)



Rationalizable Strategies

- Strategies are “rationalizable” if they could *conceivably* be played when
 1. players are rational and
 2. rationality is common knowledge
- Suppose each player has a unique rationalizable strategy. Then these strategies form a Nash equilibrium.

How to Find Rationalizable Strategies

- If a strategy is strictly dominated for some player, eliminate it
- Repeat, eliminating any strictly dominated strategies in reduced game
- All strategies that remain when you are finished are rationalizable

Example: Tourists & Natives

- Two bars (bar 1, bar 2) compete
- Can charge price of \$2, \$4, or \$5
- 6000 tourists pick a bar randomly
- 4000 natives select the lowest price bar

- Example 1: Both charge \$2
 - each gets 5,000 customers
- Example 2: Bar 1 charges \$4, Bar 2 charges \$5
 - Bar 1 gets $3000 + 4000 = 7,000$ customers
 - Bar 2 gets 3000 customers

Tourists & Natives

		Bar 1		Bar 2	
		\$2	\$4	\$4	\$5
Bar 1	\$2	10 , 10	14 , 12	14 , 15	
	\$4	12 , 14	20 , 20	28 , 15	
	\$5	15 , 14	15 , 28	25 , 25	

in thousands of dollars

Successive Elimination of Dominated Strategies

		Bar 2					
		\$2		\$4		\$5	
Bar 1	\$2	10 , 10	14 , 12	14 , 15			
	\$4	12 , 14	20 , 20	28 , 15			
	\$5	15 , 14	15 , 28	25 , 25			

		Bar 2			
		\$4		\$5	
Bar 1	\$4	20 , 20	28 , 15		
	\$5	15 , 28	25 , 25		

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Some Prototypical Games

- Prisoner's Dilemma
- Loyal Servant
- Hunter and Hunted
- price war
- defensive innovation
- audits, bluffing

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Online Game #4

Monitoring Game

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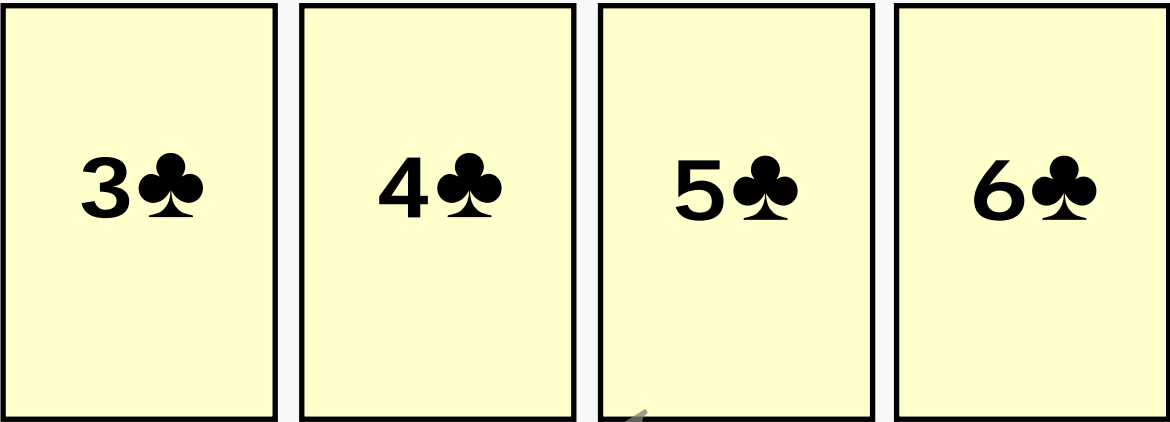
In-Class Game

Bluffing Game

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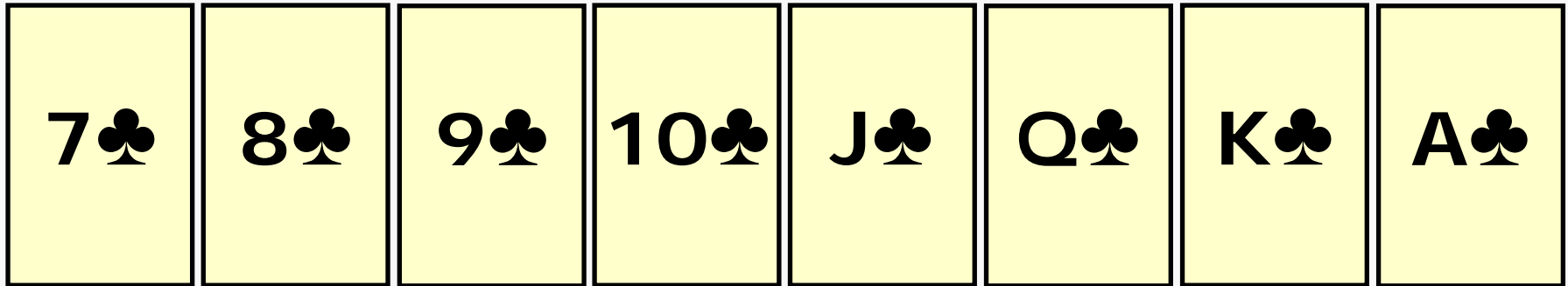
Bluffing in Poker: Set-Up

Player A's hand prior to getting 5th card



- Player A will be drawing on an inside straight flush
- Player A will have the best hand if:
 - flush (another club: 9 cards total) or
 - straight (any 2 or 7: additional 6 cards)

Winning Cards



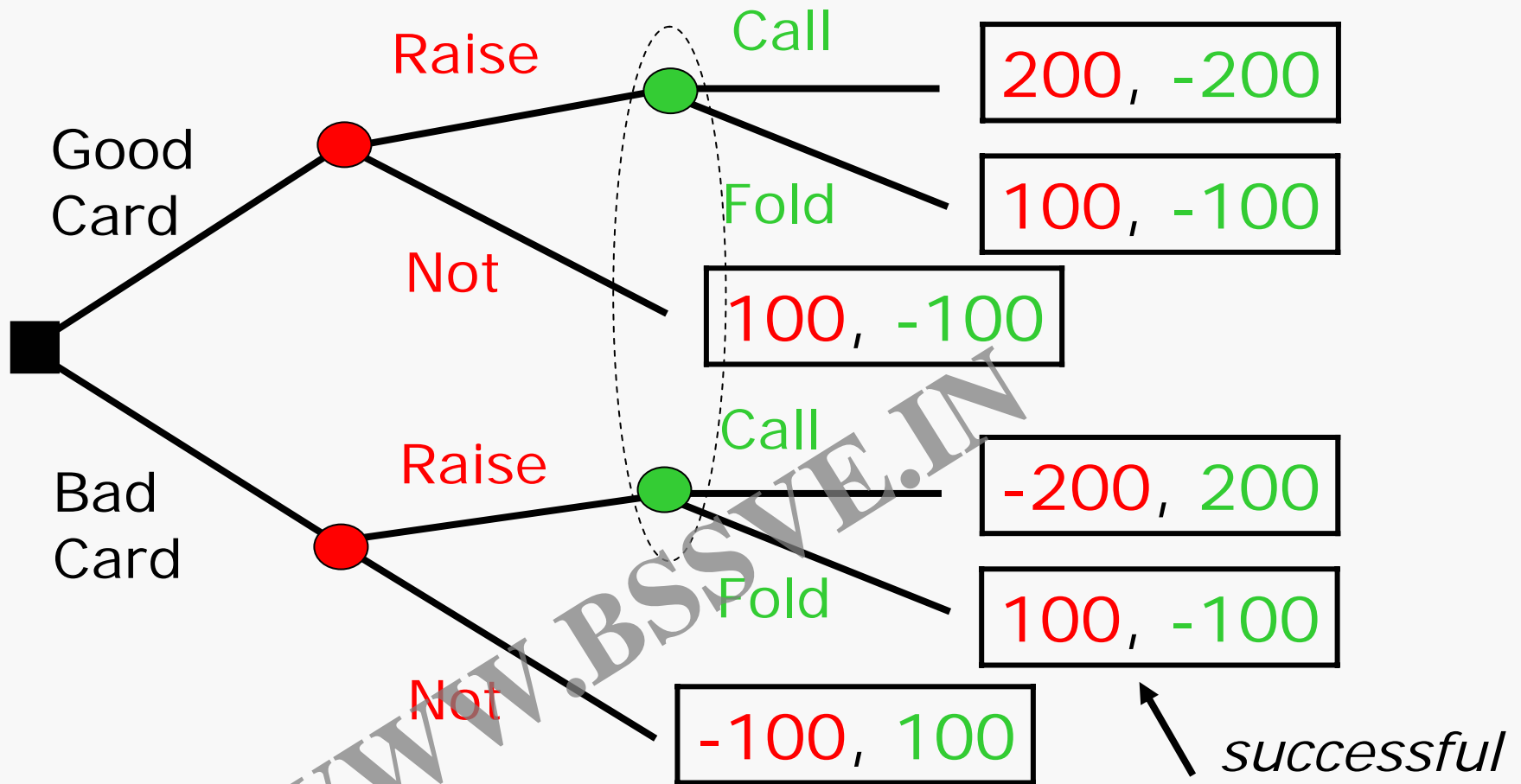
(Road sign: "Deuce of Clubs Ave.")

(Sign: "City of Show Low, AZ")

Bluffing Game: Rules

- Each player has put \$100 into the pot
- After receiving the fifth card, player A will either **Raise \$100** or **Not**
- If Raise, Player B then either **Calls** (adds \$100 more) or **Folds** (automatically losing \$100 already in pot)
- Player A wins the pot if *either* A gets winning card *or* B folds

Bluffing Game: Rules

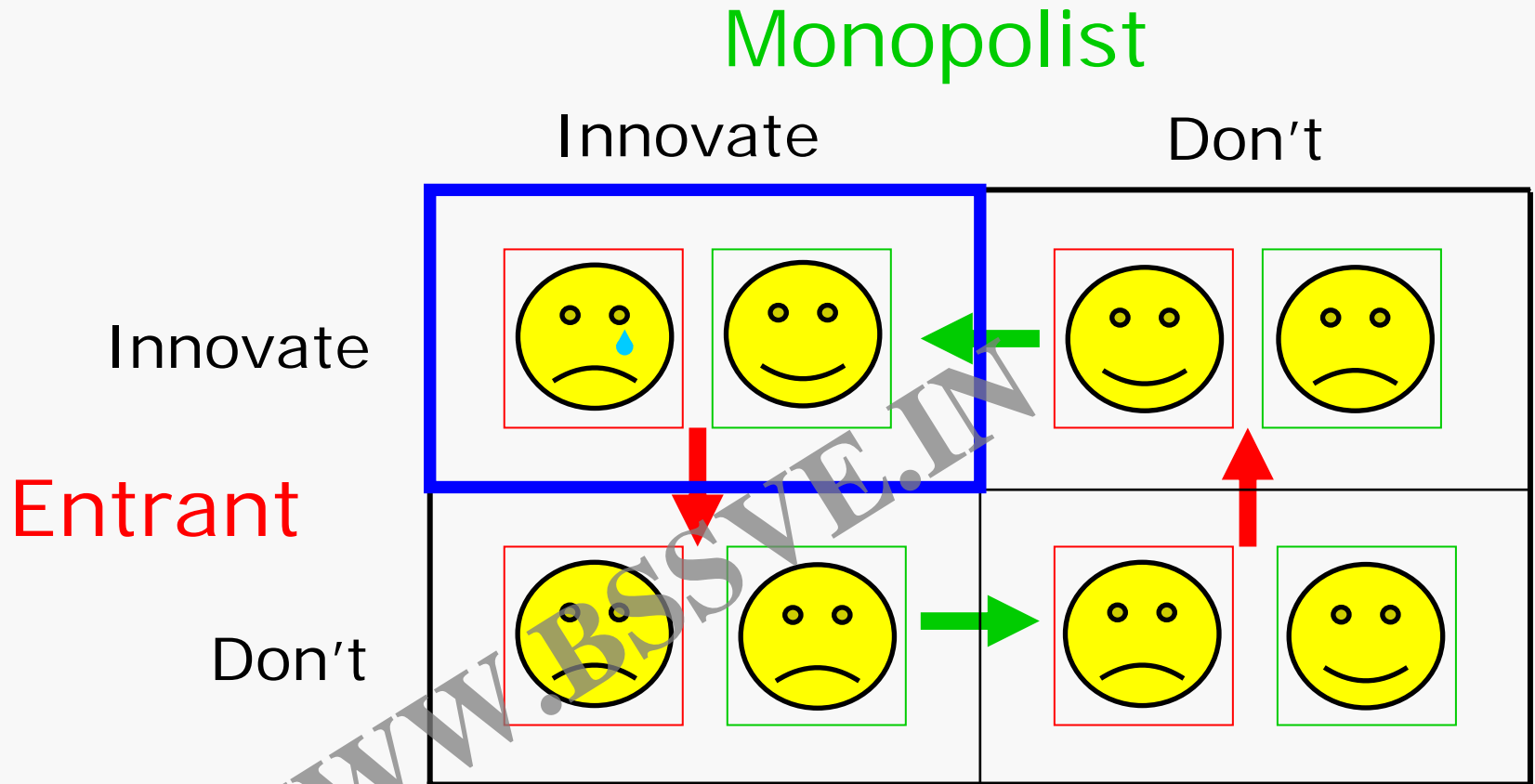


Play Bluffing Game!

- Pair up with a neighbor.
- Player A will be given a playing card
- After that, communication allowed
 - Players A,B may say (or show) anything they want to each other

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Defensive Innovation Case II: Entrant Wouldn't Innovate



Hunter and Hunted Game*

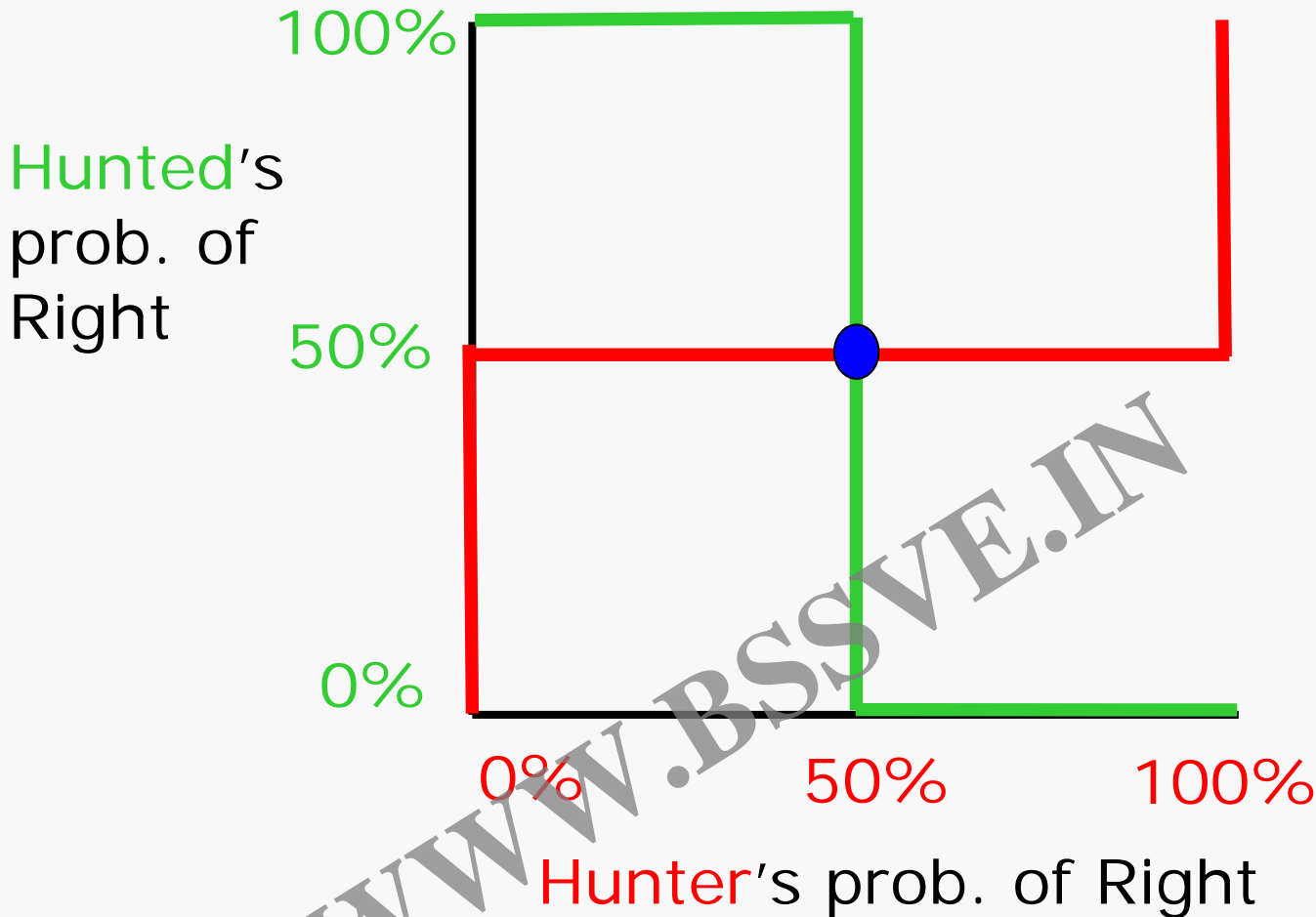
		Hunted	
		Left	Right
Hunter	Left	(2, -2)	(-2, 0)
	Right	(0, 2)	(0, 0)

■ Key features:

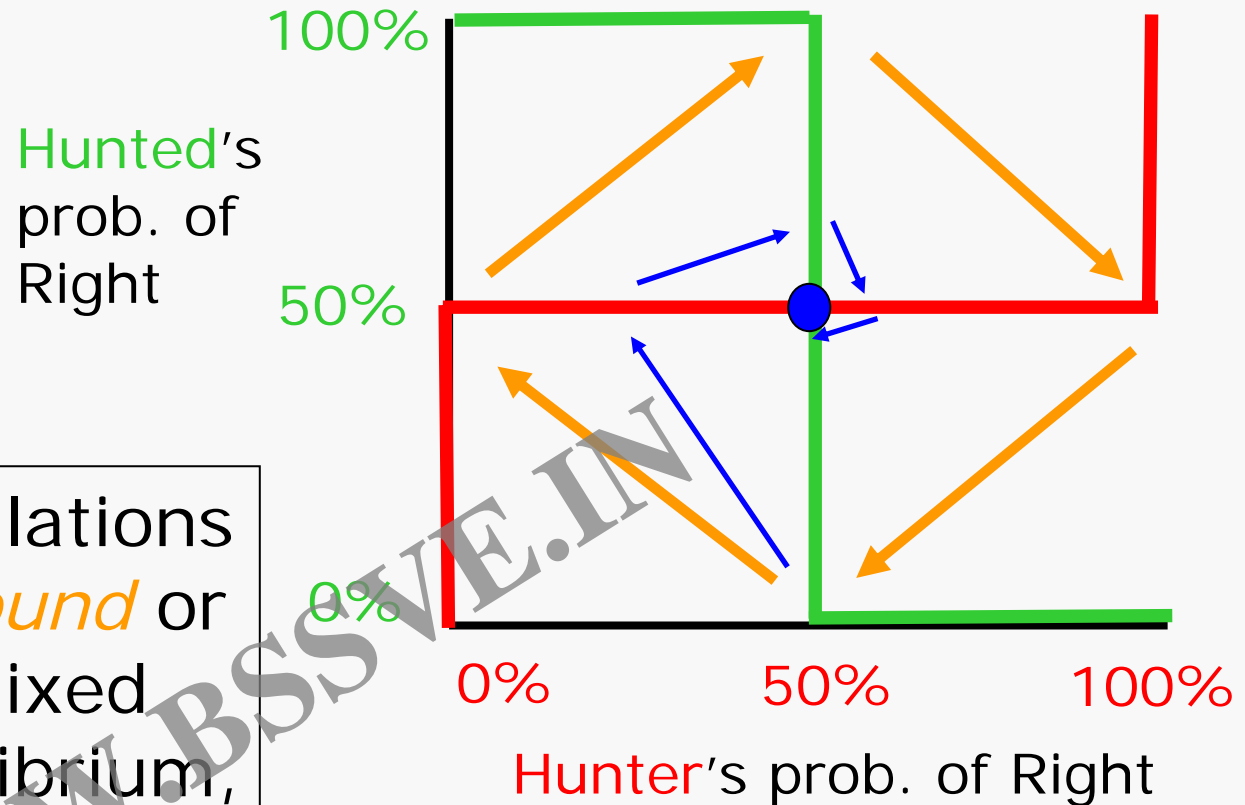
- Hunter wants to “catch”; Hunted wants to “avoid”

*Called “Attack & Defend” game in textbook

Reaction Curves in Hunter and Hunted Game



Evolution in Hunter and Hunted Game



Evolving populations may *cycle around* or *fall into* the mixed strategy equilibrium, depending on details

Side-Blotched Lizard



Summary

- Recognize dominant strategies
 - Prisoners' Dilemma
- Take others' (ir)rationality into account
 - Loyal Servant Game
- Mixing can be the right way to play
 - Hunter and Hunted Game
- *Next time*: more on evolution and introducing sequential moves

Online Game #3 (Entrant Game)

- Play Online Game #3 prior to midnight before next lecture.
- Note: We are *not* playing the games in their numerical order!!

Lecture 4 Game Plan

- February Madness
- Games with multiple Nash equilibria
 - ... which equilibrium does evolution select?
 - ... mixed strategies or pure strategies?
- Sequential move games
 - ... escaping from Annoying Servant Game

Mixed Strategies

“Ni bhionn an rath ach mar a mbionn an smacht”

“There is no luck except where there is discipline”

- old Irish proverb

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Analysis of Bluffing Game

- You get Good Card 15/48, about 1/3
- What do you do with Bad Card?
 - If you *never raise*, player B will always Fold when you have a Good Card.
 - get +100 when Good, -100 when Bad
 - average payoff about -33
 - If you *always raise*, player B will always Call you on it (even worse!)
 - get + 200 when Good, -200 when Bad
 - average payoff about -67

How Often to Raise in Eqm?

- Need to Raise enough for Player B to be indifferent between Fold and Call
 - B gets -100 if Folds
 - B gets either -200 or $+200$ if Calls
 - By Call, B “risks 100 to gain 300” relative to Fold
 - So we need $\text{Prob}(\text{Bluff} \mid \text{Raise}) = 25\%$
 - 15 Good Cards so we Bluff on 5 Bad Cards
 - So, Raise with 5/33 Bad Cards
 - When 1/3 chance of Good Card, Bluff with prob. 1/6

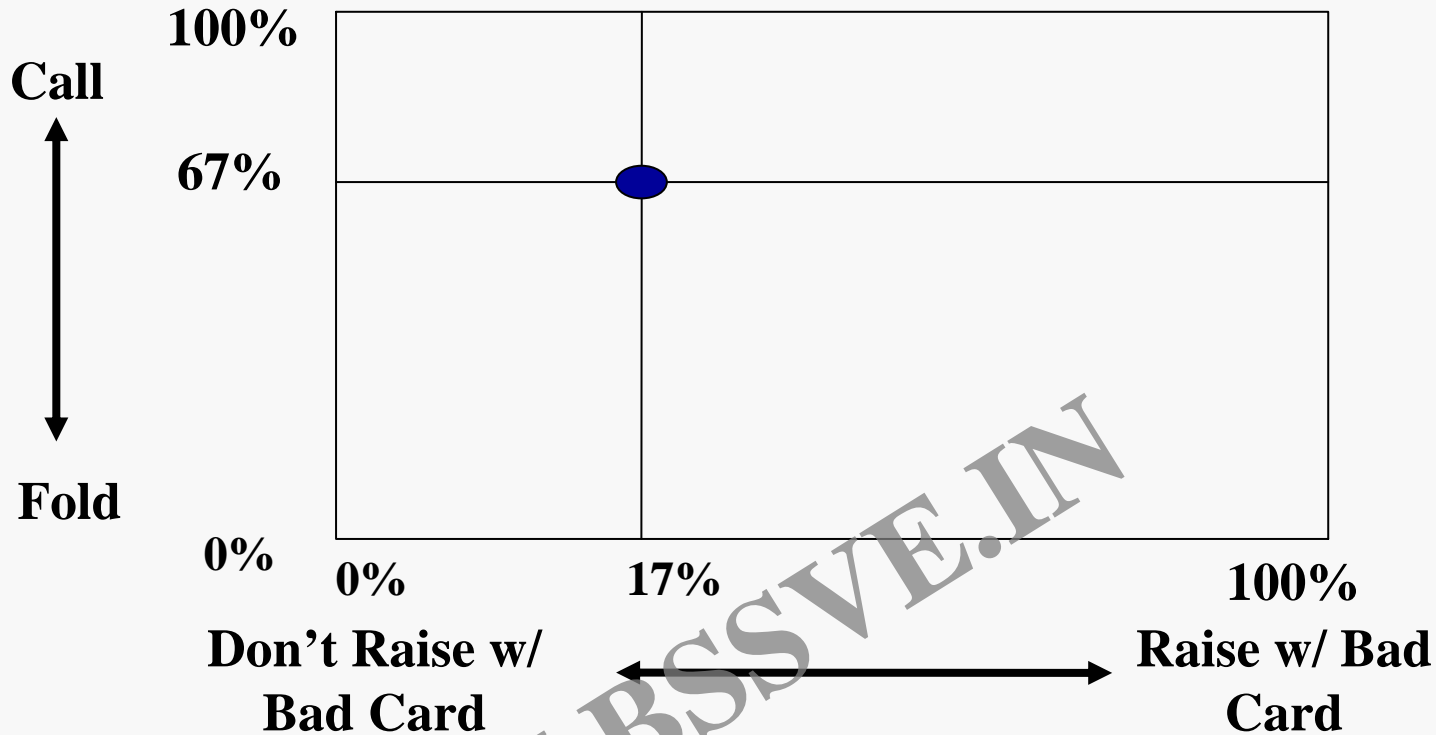
How Often to Fold in Eqm?

- Need to Fold enough for Player A to be indifferent between Raise and Not with Bad Card
 - A gets -100 if Not Raise
 - A gets either -200 or $+100$ if Raise
 - By raising, A "risks 100 to gain 200"
 - So we Fold 33%

Payoffs in Equilibrium

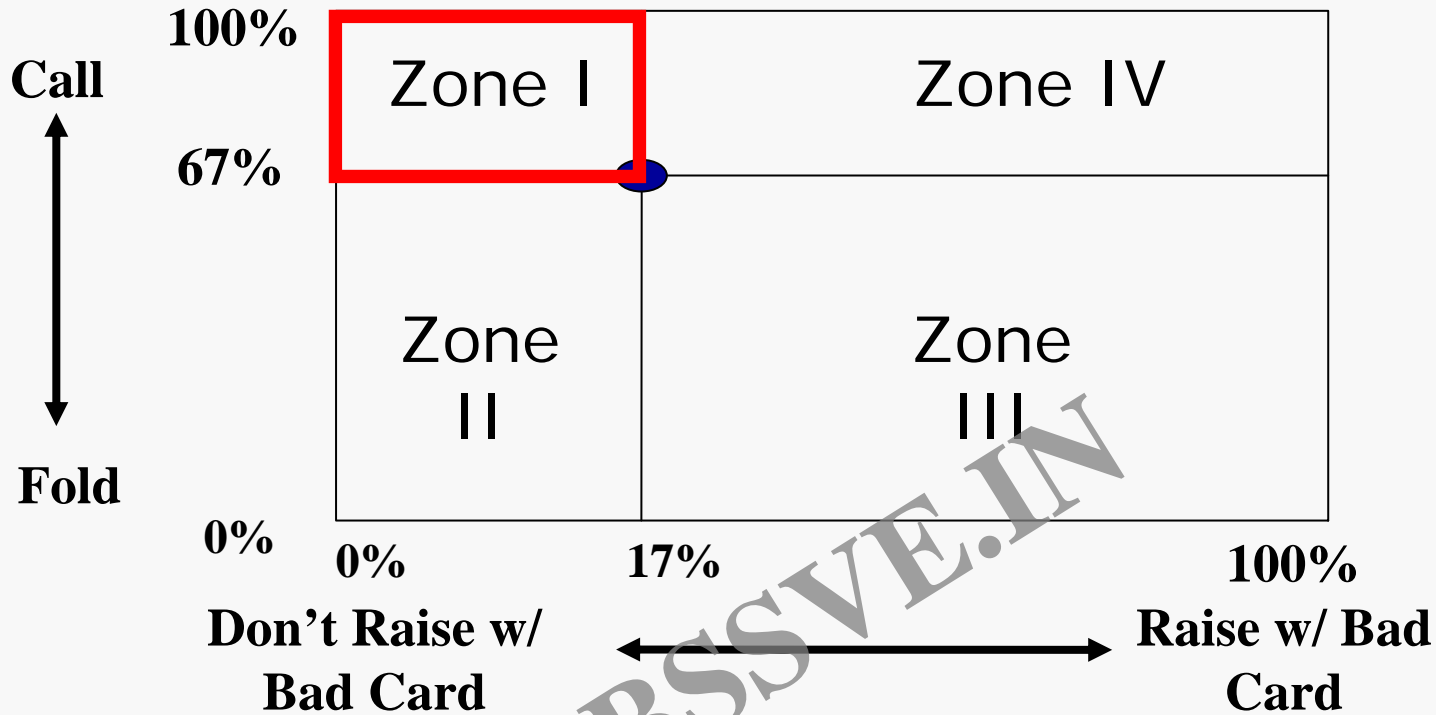
- Player B Folds 33% of time
 - Good Card $\rightarrow 33\%(+100) + 67\%(+200)$,
so get *167 when Good Card*
- ... & Player A indifferent to Raise or Not given a Bad Card
 - *-100 when Bad Card*
- Overall payoff is about **-11** for A
 - much better than always/never bluffing

Best responses in bluffing



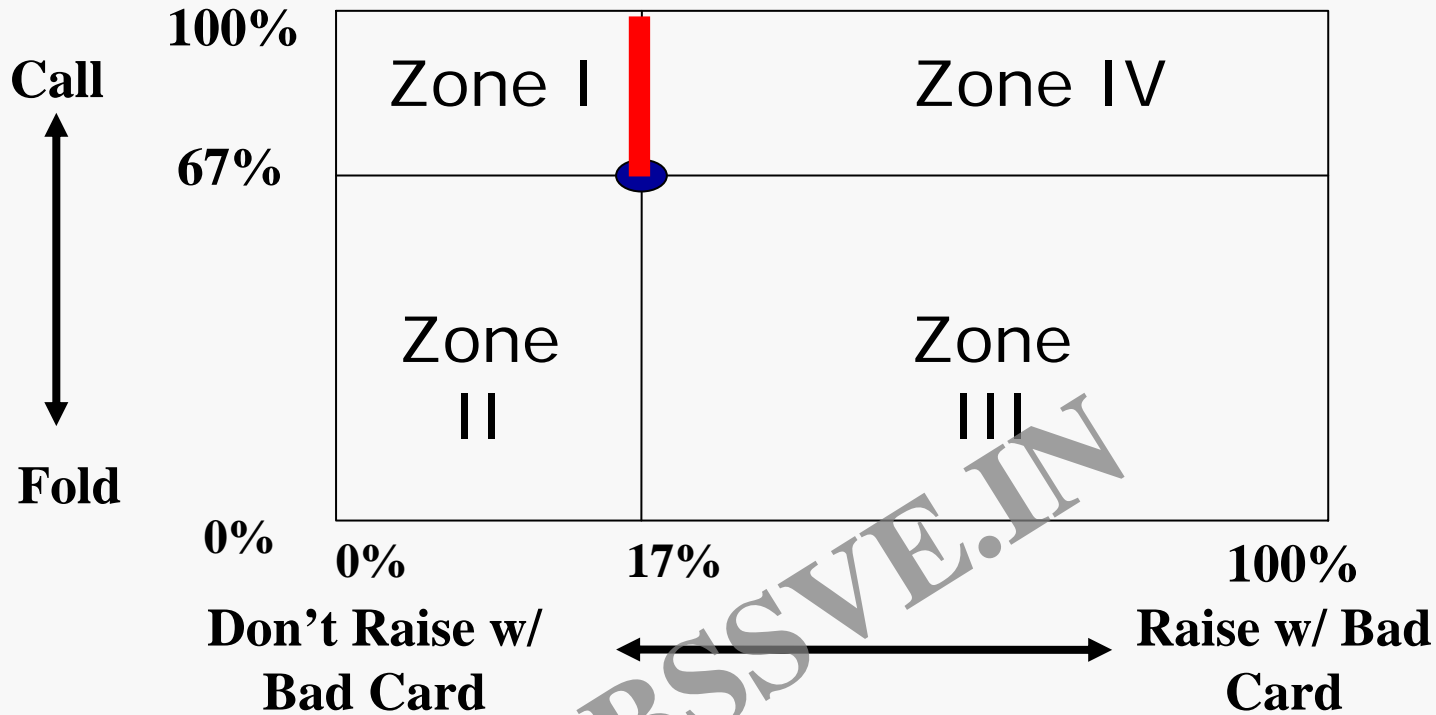
- Suppose other Raises & Folds
- What's your best response?

Best responses in bluffing



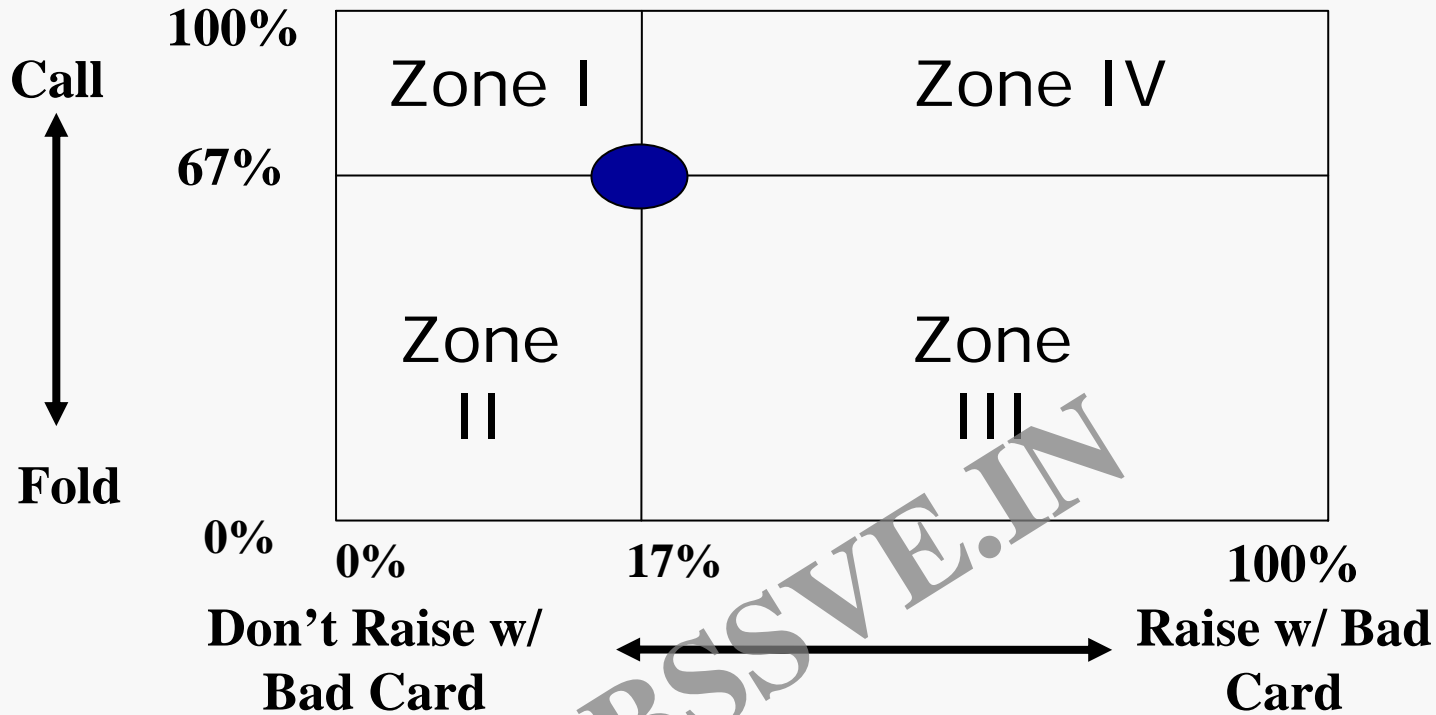
- Who will you beat if you choose strategy in Zone I?

Bluffing on a boundary



- Who will you beat if you choose on boundary of Zone I and Zone IV?

Bluffing by equilibrium



- Who will you beat if you choose the equilibrium strategy?

Some Prototypical Games

- Prisoner's Dilemma
- Loyal Servant
- Hunter and Hunted
- **Assurance**
- price war
- defensive innovation
- audits, bluffing
- **driving, cooperation**

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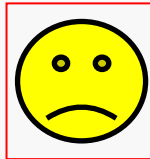





Which Side of the Road Should We Drive On?

- Map of the world showing which countries drive on the right (the majority) and which drive on the left.
- Photograph of a 1967 pamphlet explaining Sweden's change to driving on the right.
- Photograph of a car with a sign "Keep to the Right" on its dashboard reminding the driver of the new law. From Nova Scotia, 1923.

These three images are available at <http://www.brianlucas.ca> (accessed July 14, 2004).

Driving Game

Me






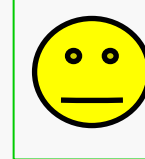



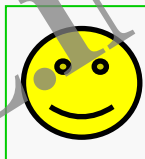

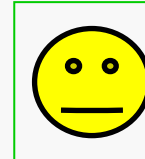






		Left	Right
You	Left	 	 
	Right	 	 

- What are the Nash equilibria in this game?

Mixed Strategies in the Driving Game

- (Left, Left) and (Right, Right) are the two *pure strategy equilibria*
- But there is also a *mixed strategy equilibrium*: each goes Left and Right half of the time
 - “Driving Chaos” is a possibility
 - You have no reason to go left vs right since *I’m driving randomly!*
 - ... so you might as well drive randomly

Mixed Strategies in the Driving Game

		Me					
		Left		Right		Random	
You	Left						
	Right						
	Random						

Assurance Game

		Column Player	
		High	Low
Row Player	High	(3, 3)	(0, 0)
	Low	(0, 0)	(1, 1)

■ Key features:

- Each wants to do the same thing as the other
- Both better off if both choose High

Assurance Game

Column Player

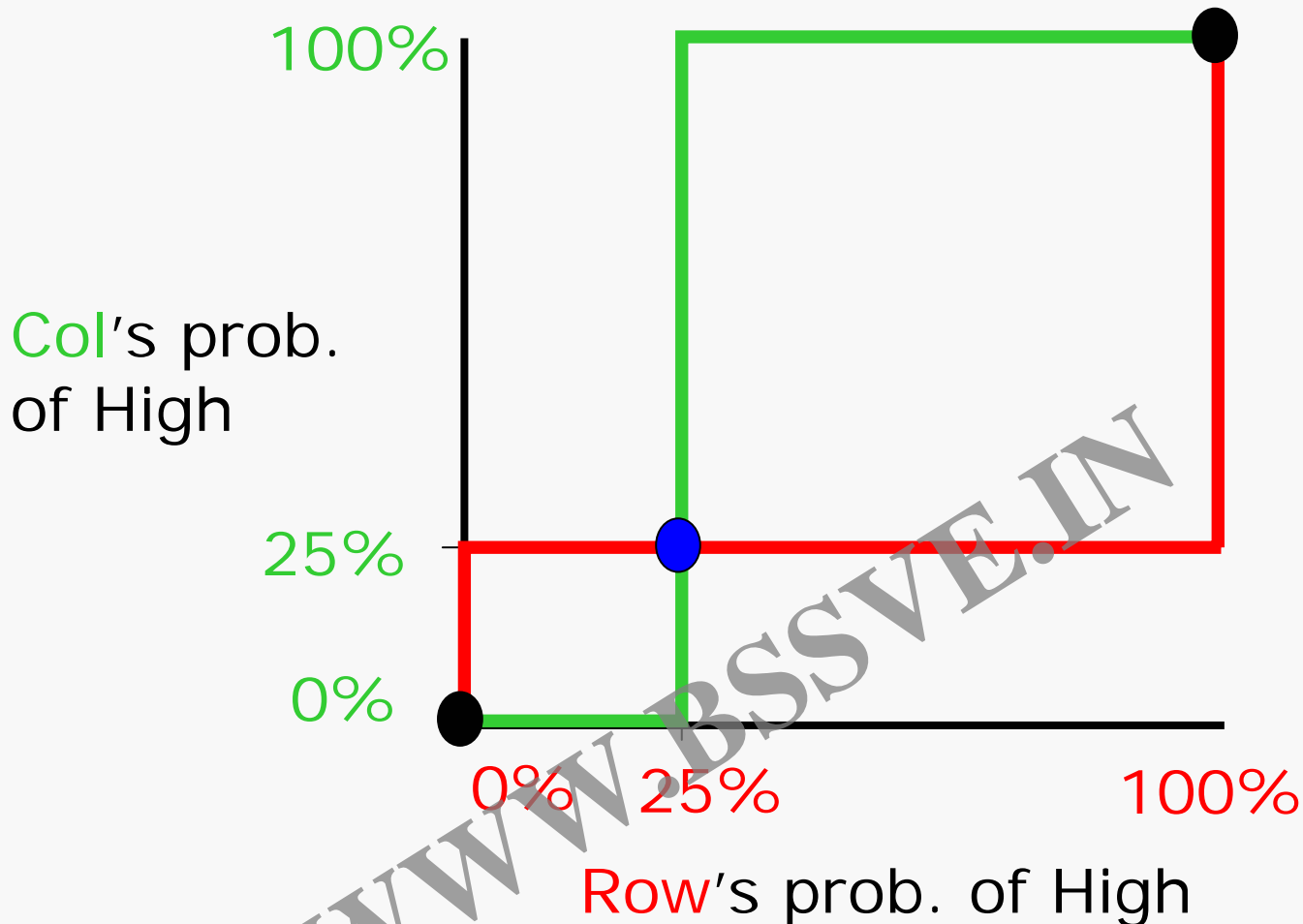
		High	Low
Row Player	High	(3, 3)	(0, 0)
	Low	(0, 0)	(1, 1)

- Two pure strategy equilibria
- PLUS a mixed strategy equilibrium in which $\text{Prob}(\text{High}) = 1/4$, $\text{Prob}(\text{Low}) = 3/4$

Mixed Strategies in the Assurance Game

- Any player who mixes between two actions **must be indifferent** between those actions
 - This requires that **the other player** mix with just the right probabilities to create this indifference
- If $\text{Prob}(\text{High}) = 1/4$ & $\text{Prob}(\text{Low}) = 3/4$ is *my* mixture, then *you* get payoff $3/4$ no matter what you do.
 - Each player adopts the better action **less often (!!)** in the mixed-strategy eqm

Reaction Curves in Assurance Game



Evolution & Stability of Play

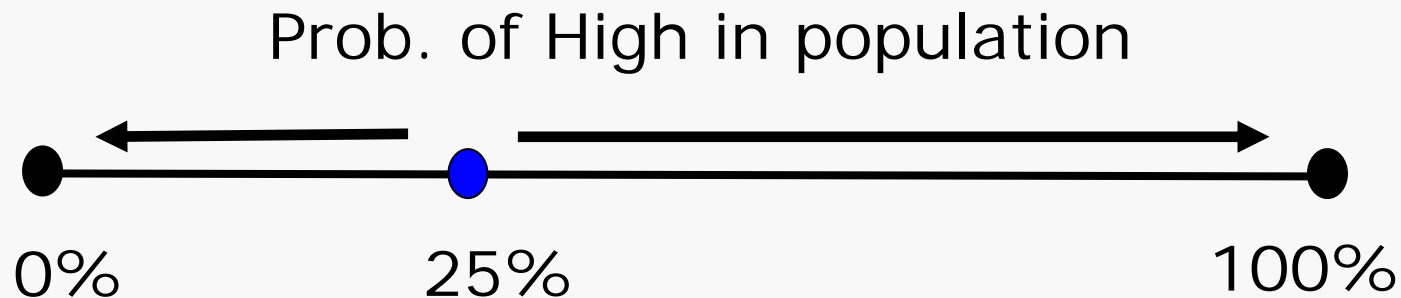
- Suppose that (1) players are “hard-wired” for either High or Low and (2) those who get higher payoffs become relatively more numerous.

- What are evolutionarily stable strategies (ESS)?
 - Any ESS must be Nash equilibrium!
 - Not all Nash equilibria are ESS

ESS in Assurance Game

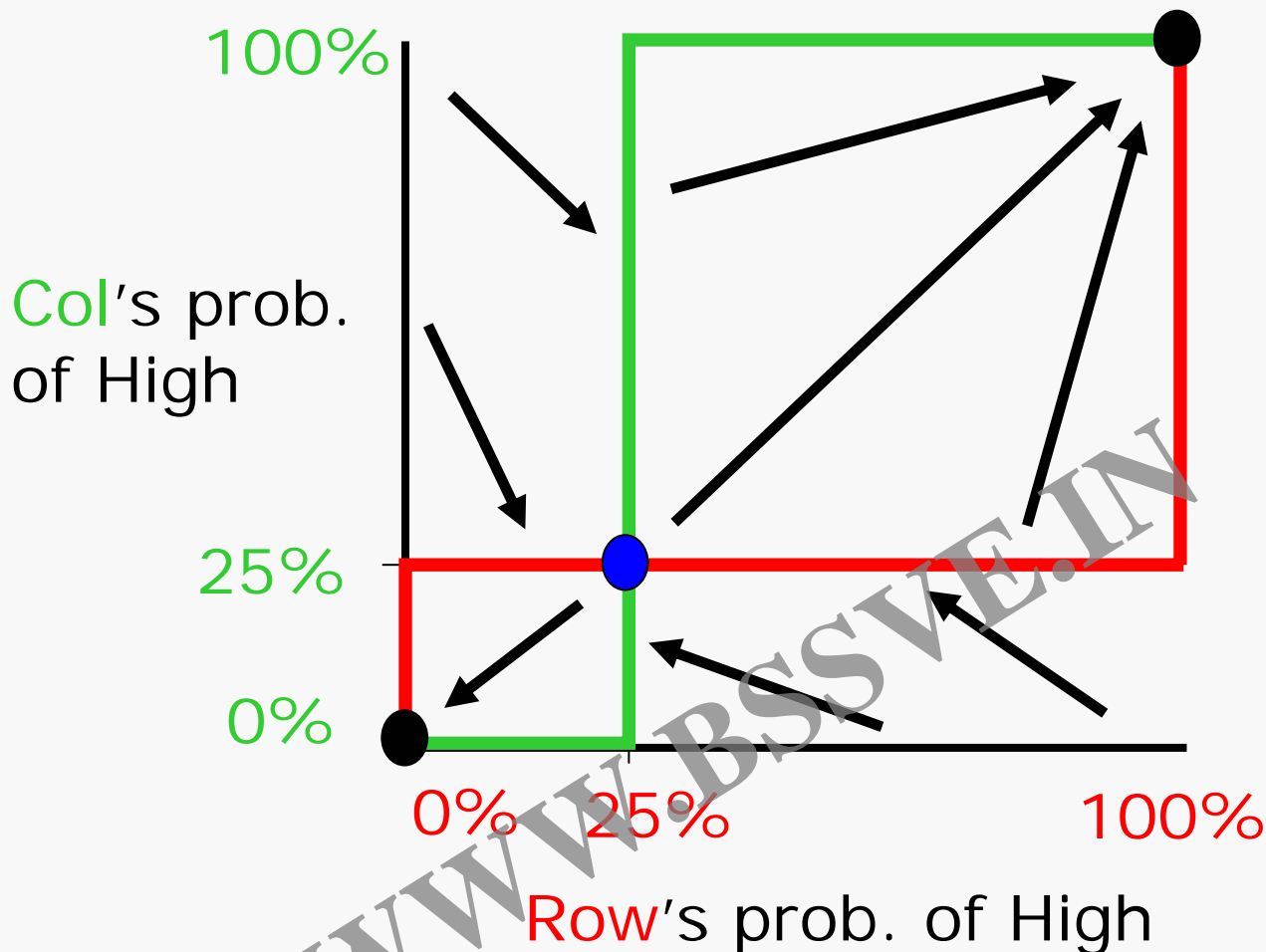
- Both pure strategy equilibria are evolutionarily stable
- The mixed-strategy equilibrium (MSE) is *not* evolutionarily stable
 - In the MSE, 25% of population plays High and 75% plays Low
 - If a few extra people are born (say) High, that will increase payoff to High and decrease payoff to Low, putting Low-people at a disadvantage
 - This will push everyone toward playing High

Evolution in Assurance Game with One Population



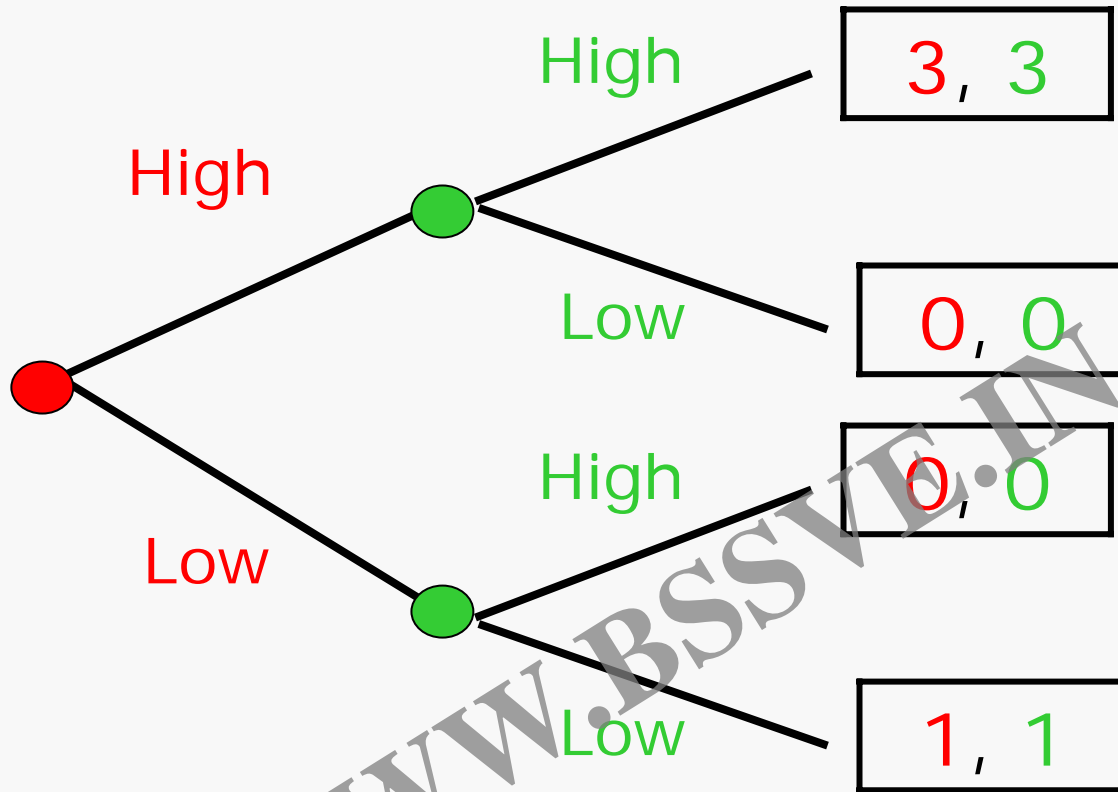
- **Row** and **Col** players are drawn from the same population
- If initial condition $< 25\%$ High-types, then evolution will push us to Low eqm, and vice versa

Evolution in Assurance Game with Two Populations



Now **Row** is drawn from "reds" and **Col** from "greens", where these populations *evolve separately*

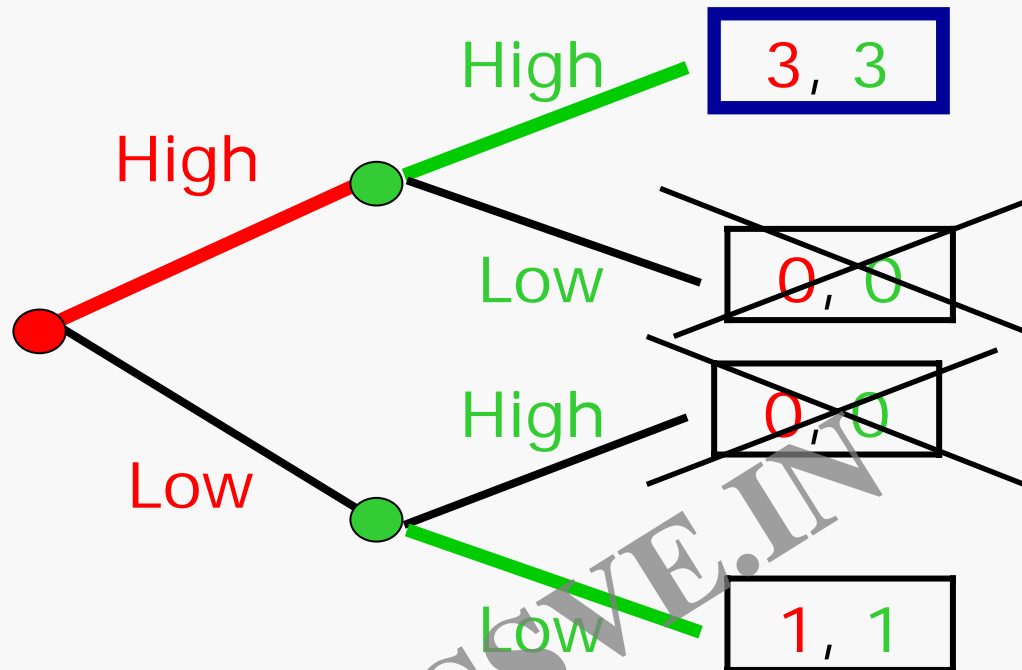
Sequential Moves in Assurance Game



How to Find Subgame-Perfect Equilibrium

- Early movers make choices assuming that later movers will make whatever choice is in their best interest
- “Rollback procedure”
 - start at the *terminal decision nodes* in the game tree, and work backwards thru the tree

Sequential Moves in Assurance Game



- Players *coordinate* on (High, High) in the subgame-perfect equilibrium

On-line Game #3

Entrant Game

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Equilibria in Sequential Move Games

- The meaning of “equilibrium” is entirely different in a sequential move game.
 - Before: Each player chooses a best response to others' fixed strategies
 - since simultaneous moves, you can't change others' choice by your choice
 - Now: Each chooses a best response to others' responsive strategies

Equilibria in Sequential Move Games

- In Assurance and Chicken Games, the outcome of the sequential-move version has been same as in a Nash equilibrium
- This need not be the case!

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Summary

- How to play in Hunter & Hunted game
 - use equilibrium probabilities as benchmark
 - assess whether other player is likely Evolution may lead to pure or mixed strategy equilibrium.

- Assurance Game
 - with and without pre-emptive moves

- *Next time*: more on commitment

Online Game #6 (New Market Game)

- Play Online Game #6 prior to midnight before next lecture.
- Note: We are *not* playing the games in their numerical order!!

Lecture 5 Game Plan

- Qwest Bond Swap
- Chicken Game
- Sequential move games
 - ... escaping from the Annoying Servant
 - ... escaping from the Prisoners' Dilemma
- More on strategic moves
 - Pre-emptive moves, threats, and promises

Qwest Bond Swap

“If Judge Chin allows the offer to go ahead, institutional investors who own bonds will find themselves in a position with **some resemblance to the classic ‘prisoners’ dilemma**’ ... If no one tendered, then Qwest would be in the same position as before the offer, and any bondholder would be no worse off. But if a lot of holders tender, those who refuse will be worse off than they were.”

- Norris, Floyd. “A Bond Swap Available Only to Big Players,” *The New York Times*, 18 December 2002.

Figure for Q1, Q2 (and Q3)

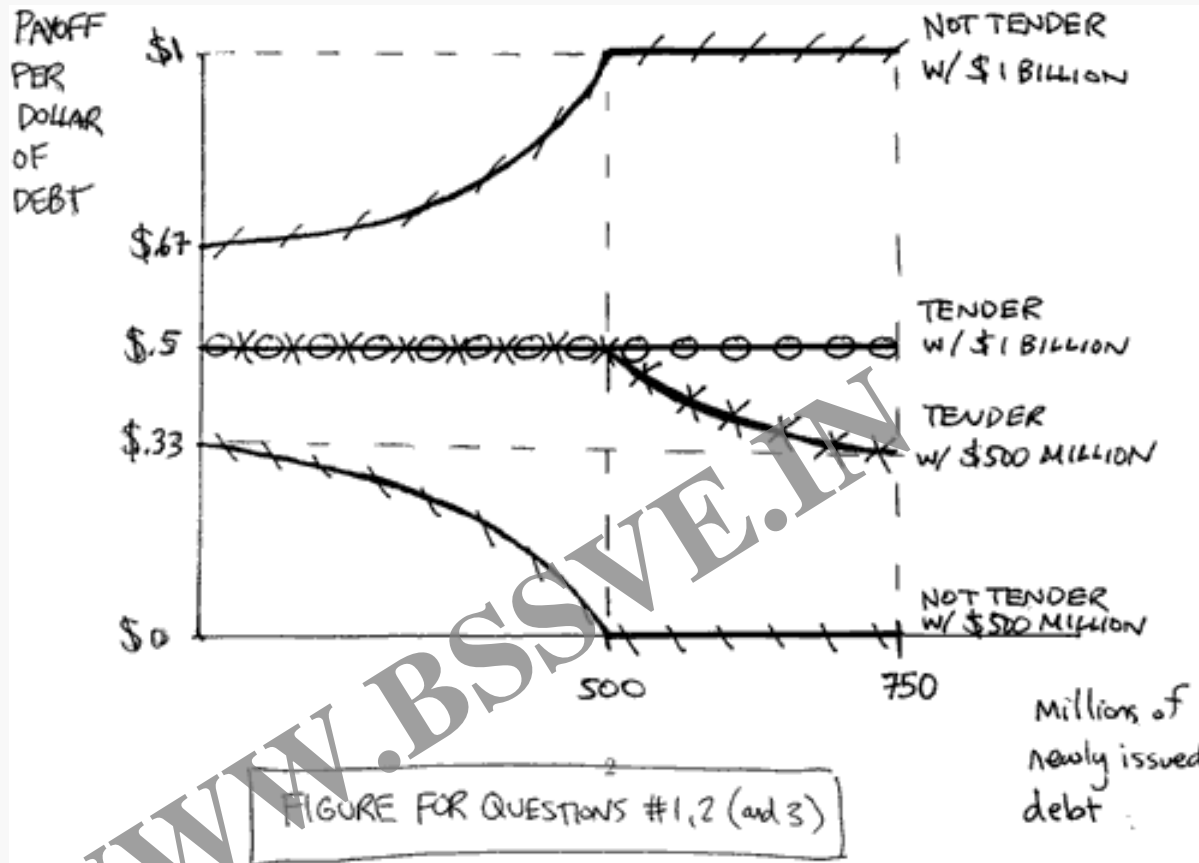
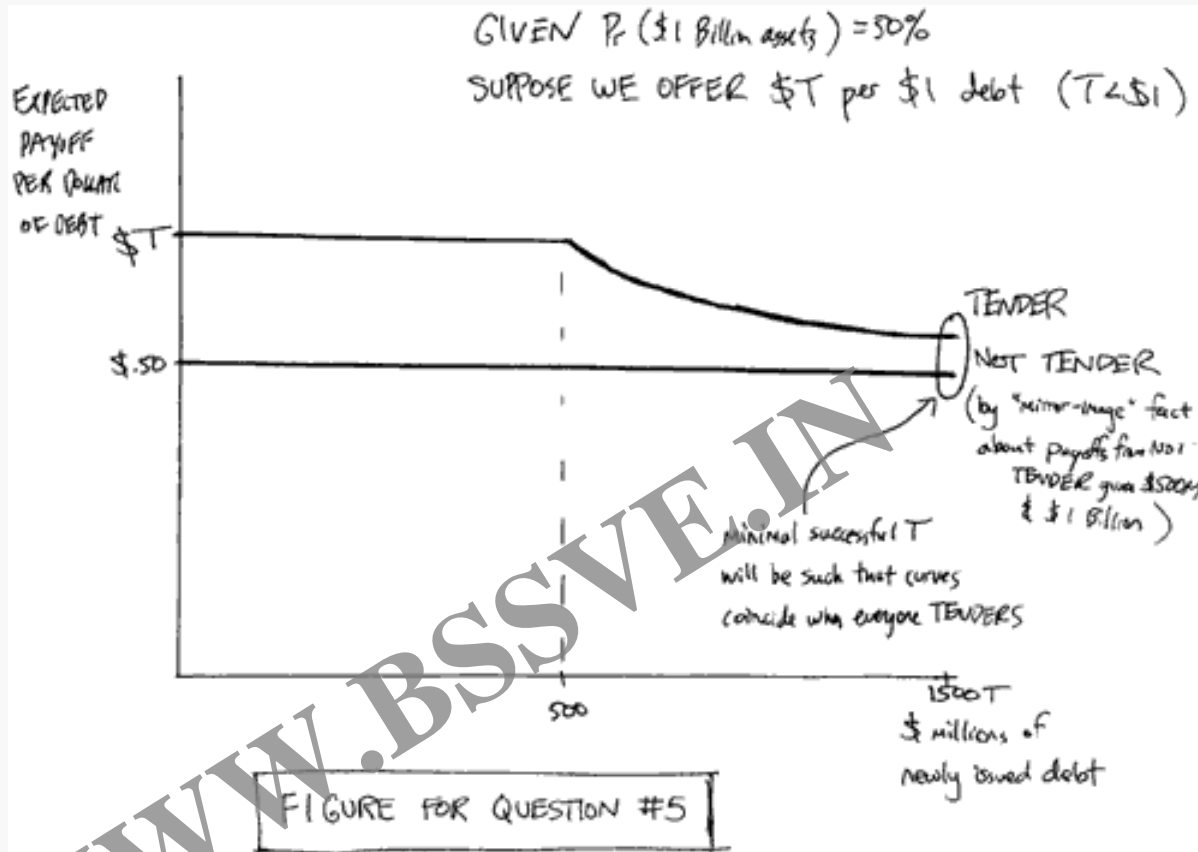


Figure for Q5



Changing the Game

"Always be wary of the superstructure of whatever situation you're in. It may just be that the whole game that you're into is something very bogus and you should get out."

- Scott Miller, Game Theory lead guitarist,
in: Woelke, Tina. "Where Have You Gone, James
Joyce? A Nation Turns Its Lolita Eyes To You."
Non-Stop Banter, December 1988.

Game Theory

- *Prominent “power paisley pop” band in 80s*
 - *formed in 1981*
 - *signed by Rational Records*
- *from first album, “Blaze of Glory”...*
 - *“All I Want is Everything”*
 - *“Bad Year at UCLA”*
 - *“Sleeping Through Heaven”*

Some Prototypical Games

- Prisoner's Dilemma
- Loyal Servant
- Hunter and Hunted
- Assurance
- **Chicken**
- price war
- defensive innovation
- audits, bluffing
- driving, cooperation
- **negotiation**

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On-Line Game #6

New Market Game

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Negotiation Game

- Used car dealer says that the lowest possible price is \$20,000
 - actual cost is \$17,000
- Buyer says that the highest possible price is \$18,000
 - actual value is \$21,000
- Each player has two strategies:
"Give in" or "Not"

Negotiation Game

Dealer

		Give in	Don't
Buyer	Give in	(2,2)	(1,3)
	Don't	(3,1)	(0,0)

in thousands of dollars

- How would you play this game?

Mixed Strategies in the Negotiation Game

- (Give In, Don't) and (Don't, Give In) are the two *pure strategy equilibria*
- There is also a *mixed strategy equilibrium*: Prob(Give In) = 50%
 - failure to agree 25% of the time!
- Are any of these equilibria evolutionarily stable?

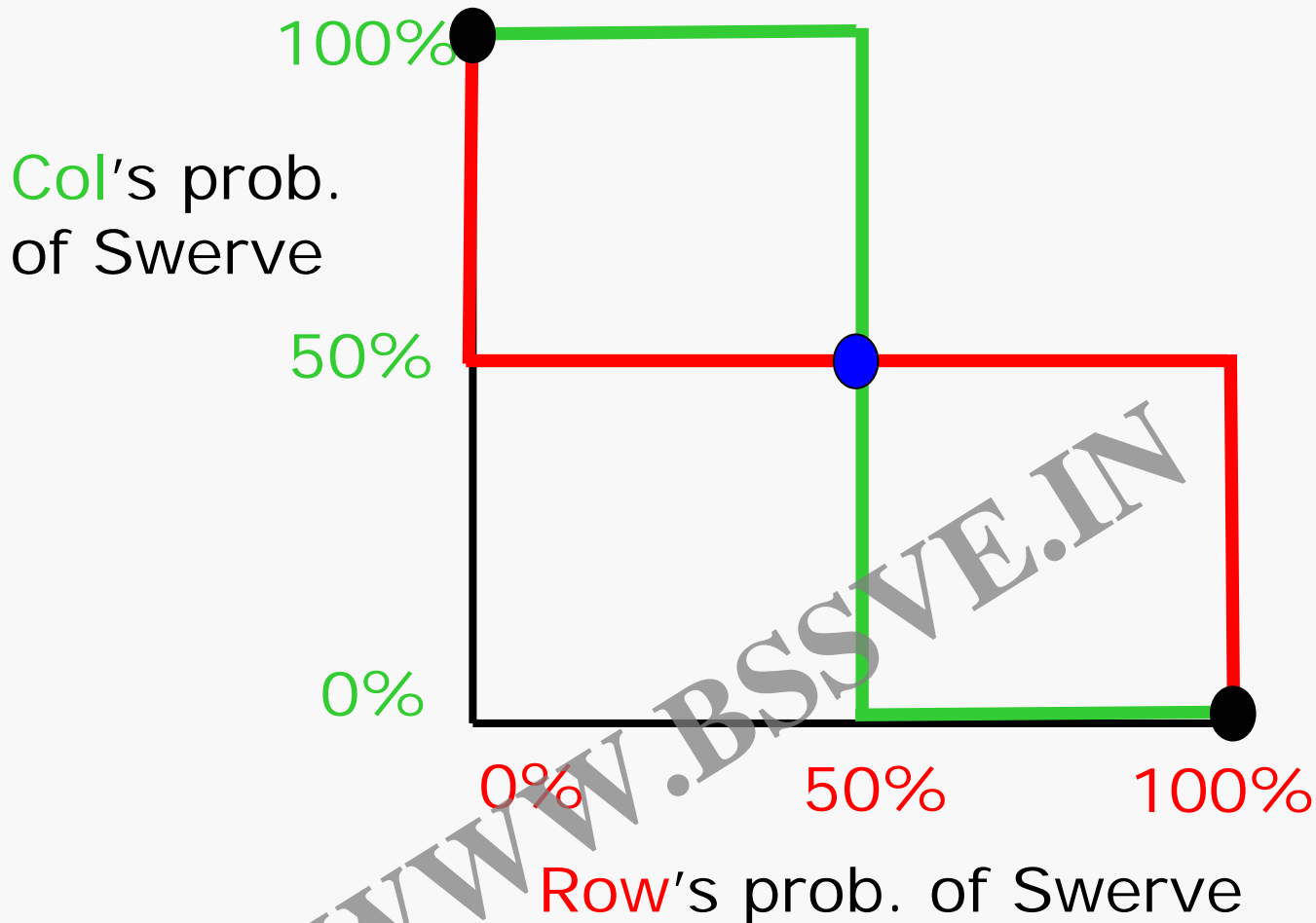
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Chicken Game

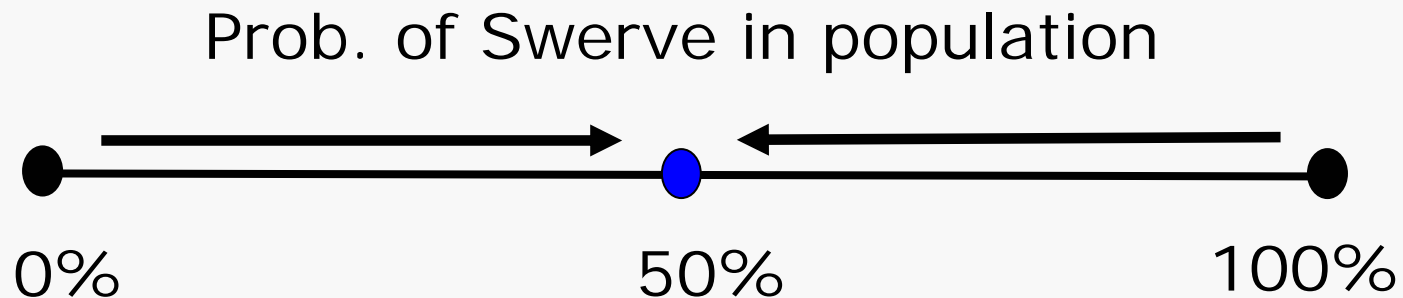
		Column Player	
		Swerve	Don't
Row Player	Swerve	(2,2)	(1,3)
	Don't	(3,1)	(0,0)

- Key features:
 - Each wants the *other* to choose Swerve
 - Both better off if both choose Swerve rather than Don't

Reaction Curves in Chicken Game

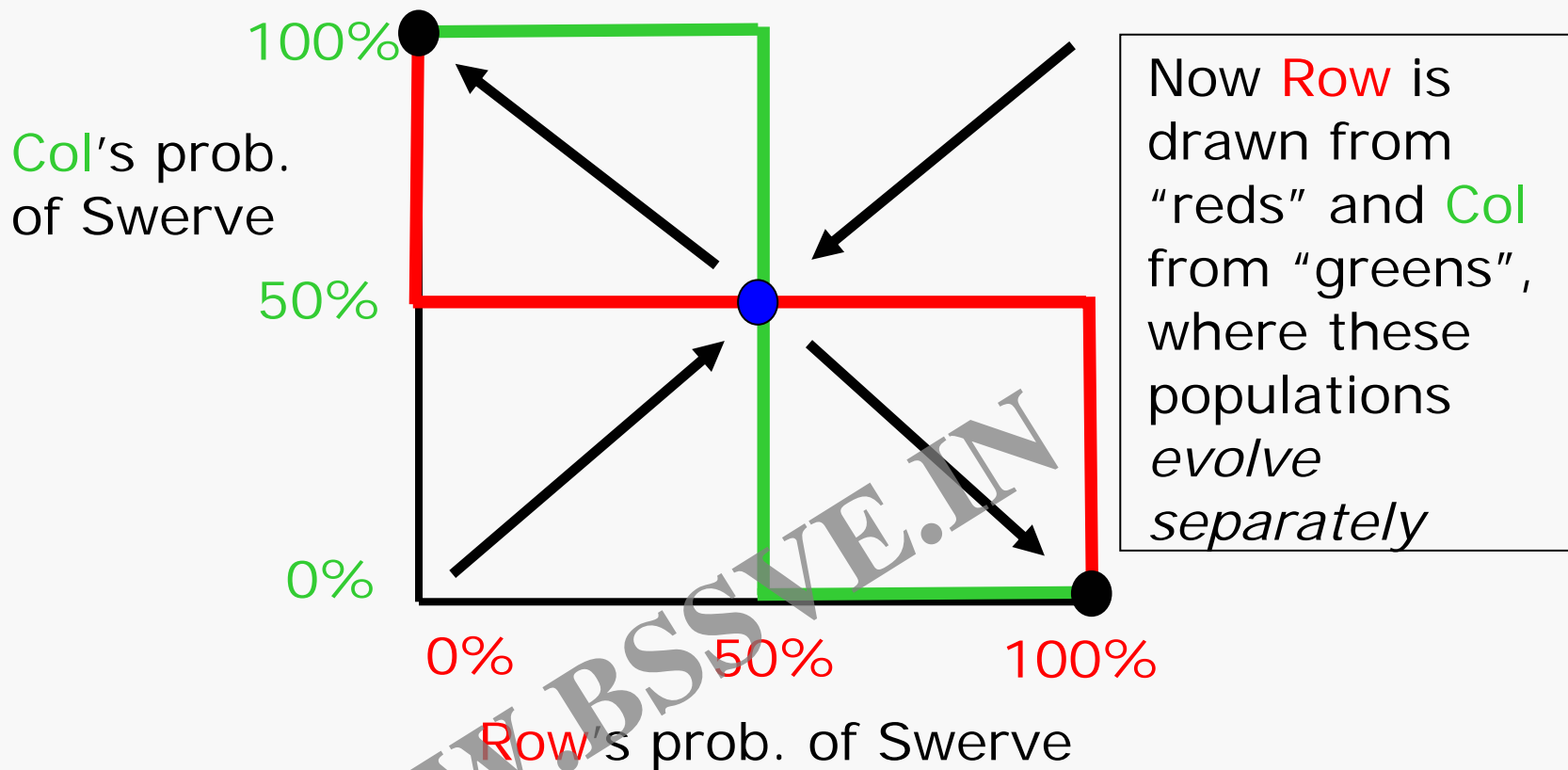


Evolution in Chicken Game with One Population



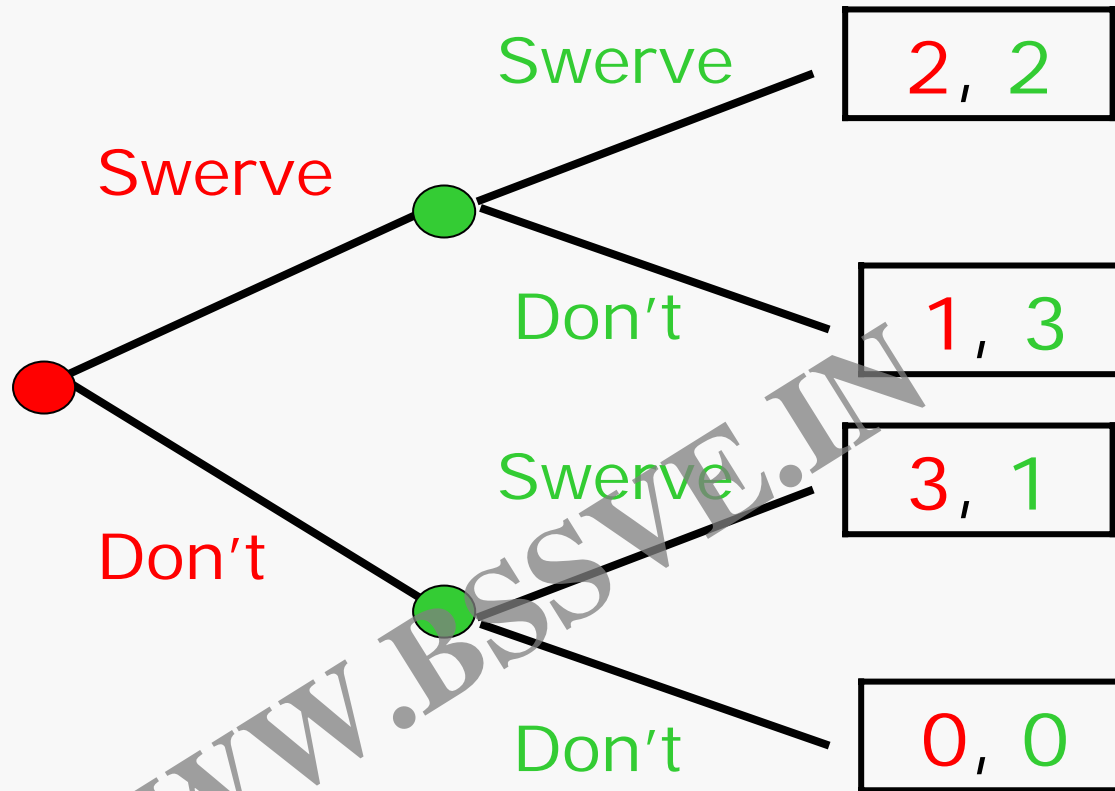
- If initial condition $< 50\%$ Swervers, then *non-Swervers* are relatively better off.
- *Only the mixed strategy equilibrium* is evolutionarily stable with one population

Evolution in Chicken Game with Two Populations

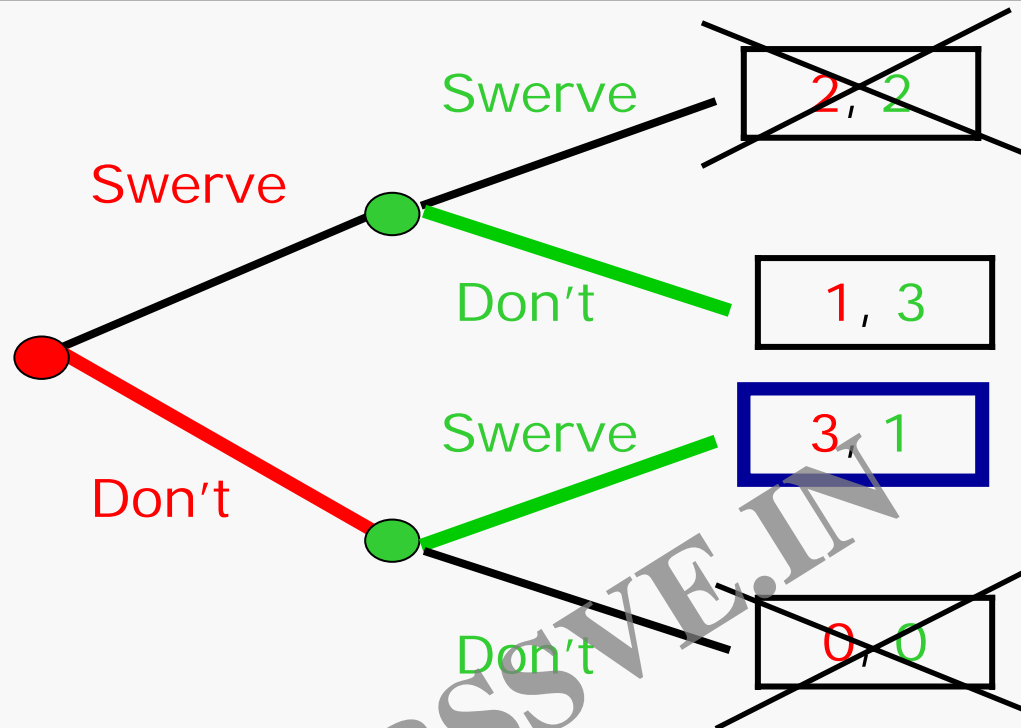


- *Only the pure strategy equilibria are evolutionarily stable with two populations*

Sequential Moves in Chicken Game



Sequential Moves in Chicken Game



- Each player prefers to be first-mover
- Being first-mover here allows you to “select your favorite equilibrium”

Sequential Games

*“Life must be understood backward,
but it must be lived forward.”*

- Soren Kierkegaard

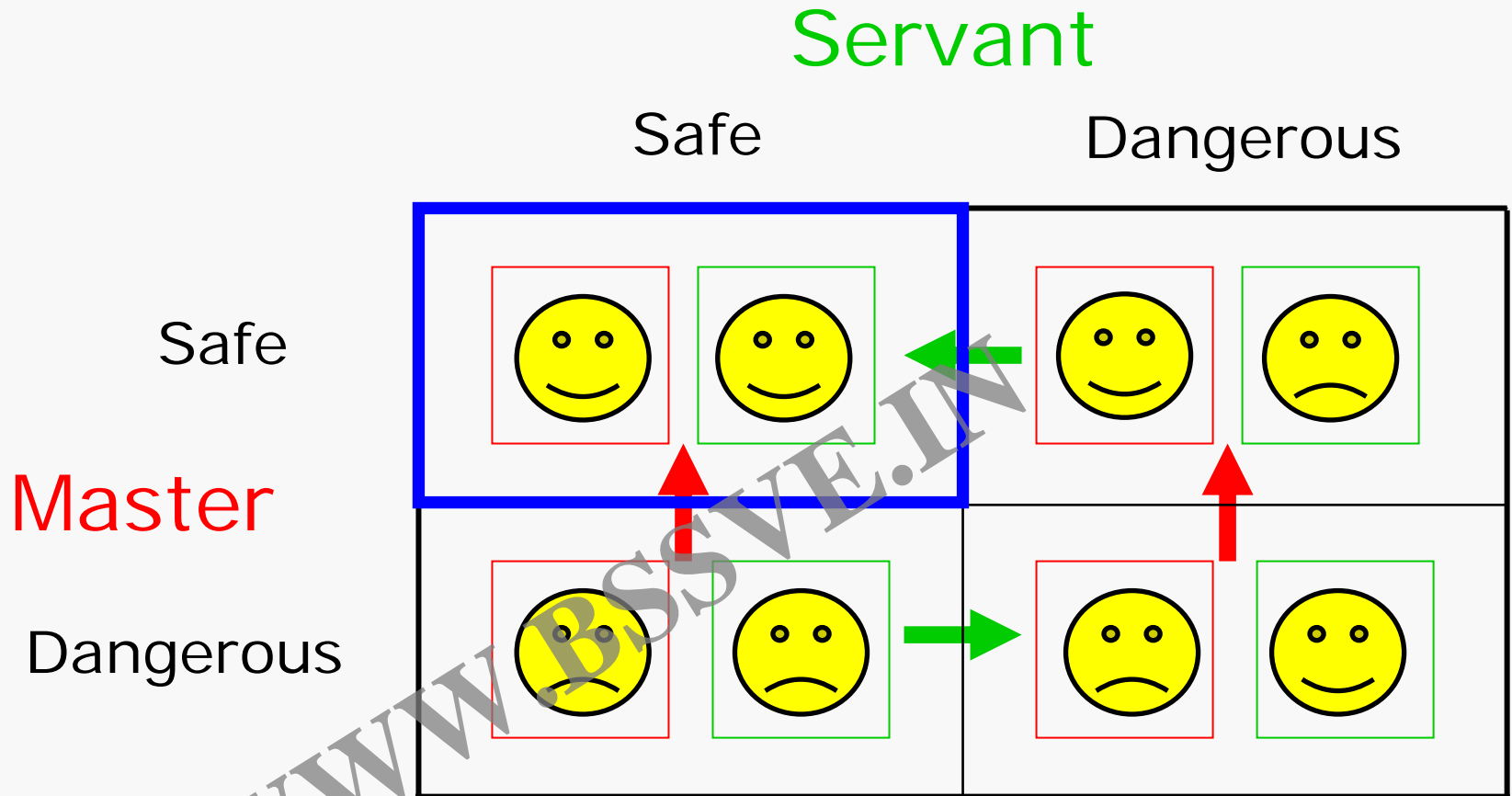
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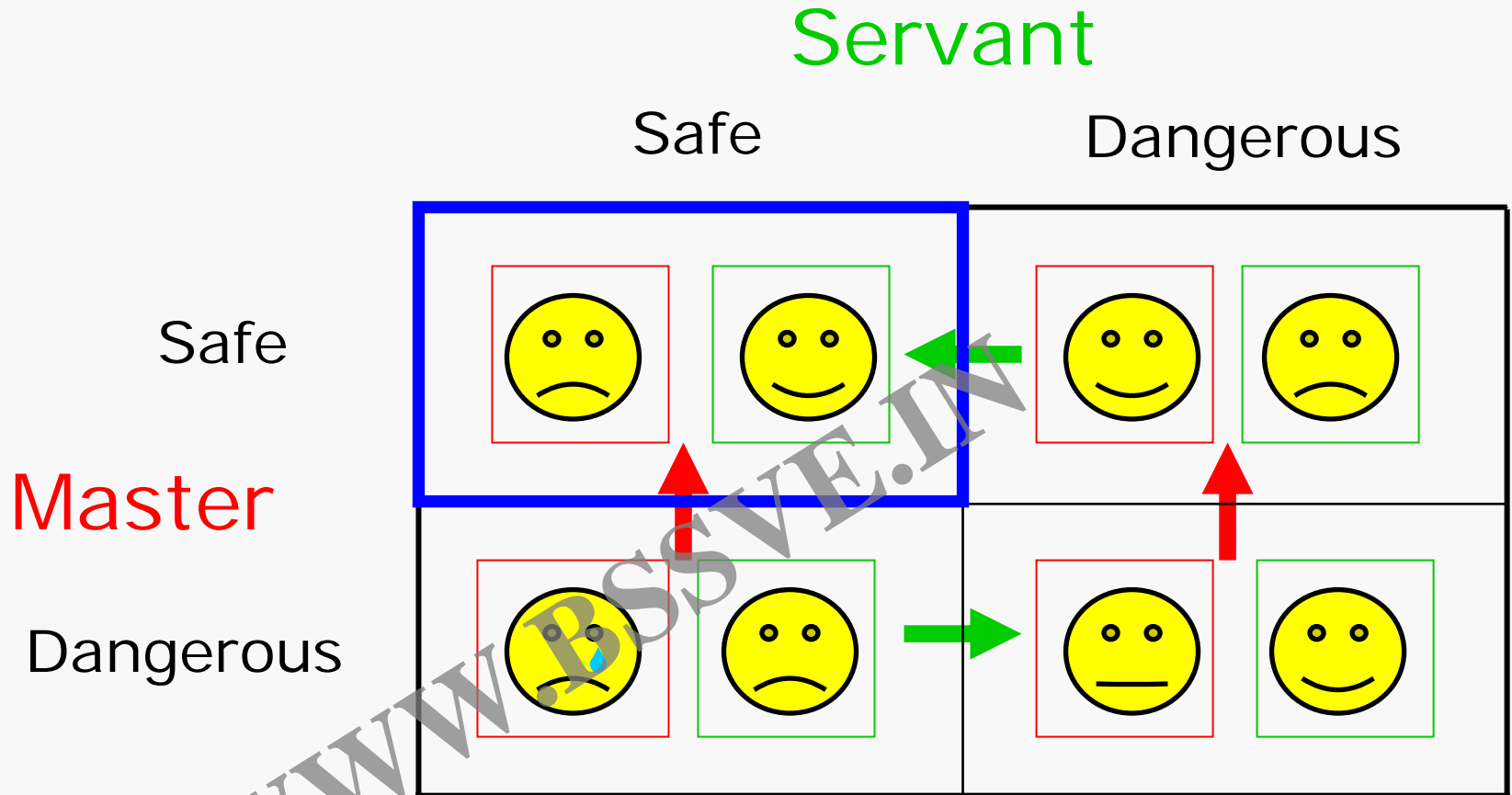
Lazy Husband Game

- Husband and wife both work long hours but can't afford a housekeeper
- Wife happy to do housework if Husband also does, but not if Husband shirks
- Husband has dominant strategy to shirk
- Both prefer that both do housework than that both shirk

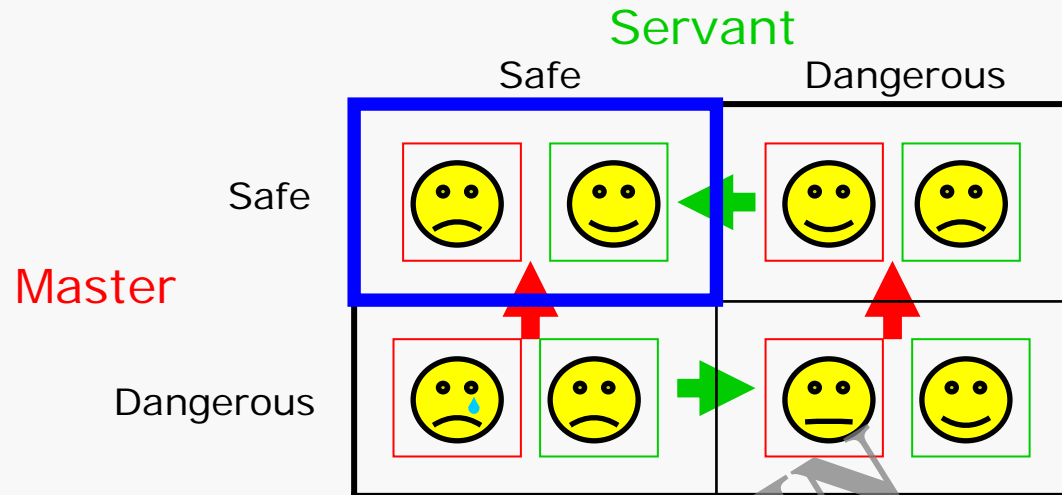
Loyal Servant Game



Special Case: Annoying Servant Game



"Story" behind Annoying Servant Game



- Servant wants to be with Master
- Servant is annoying → Master likes (S,D) most
- Dangerous route not passable alone → Master prefers (S,S) over (D,S)
- Servant might "accidentally" fall off the cliff → Master prefers (D,D) over (S,S)

Lazy Husband Game

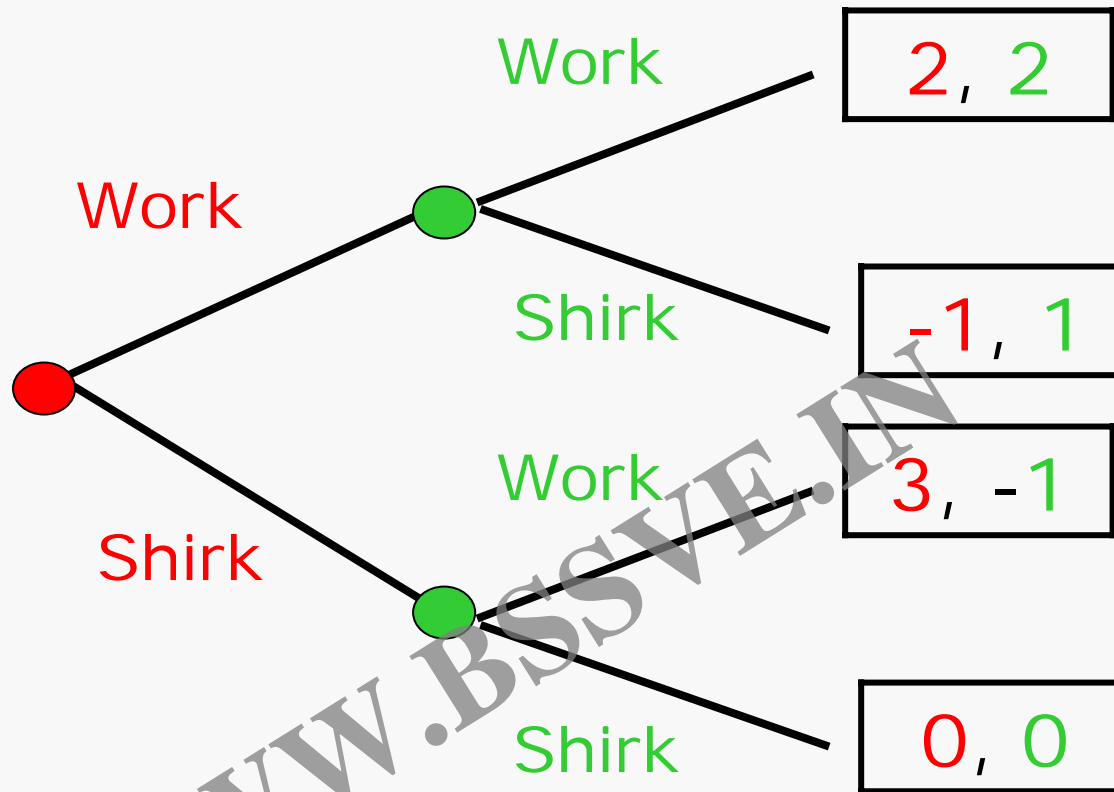
Wife

		Work	Shirk
Husband	Work	(2, 2) ←	(-1, 1) ↓
	Shirk	(3, -1) ↓	(0, 0) ↓

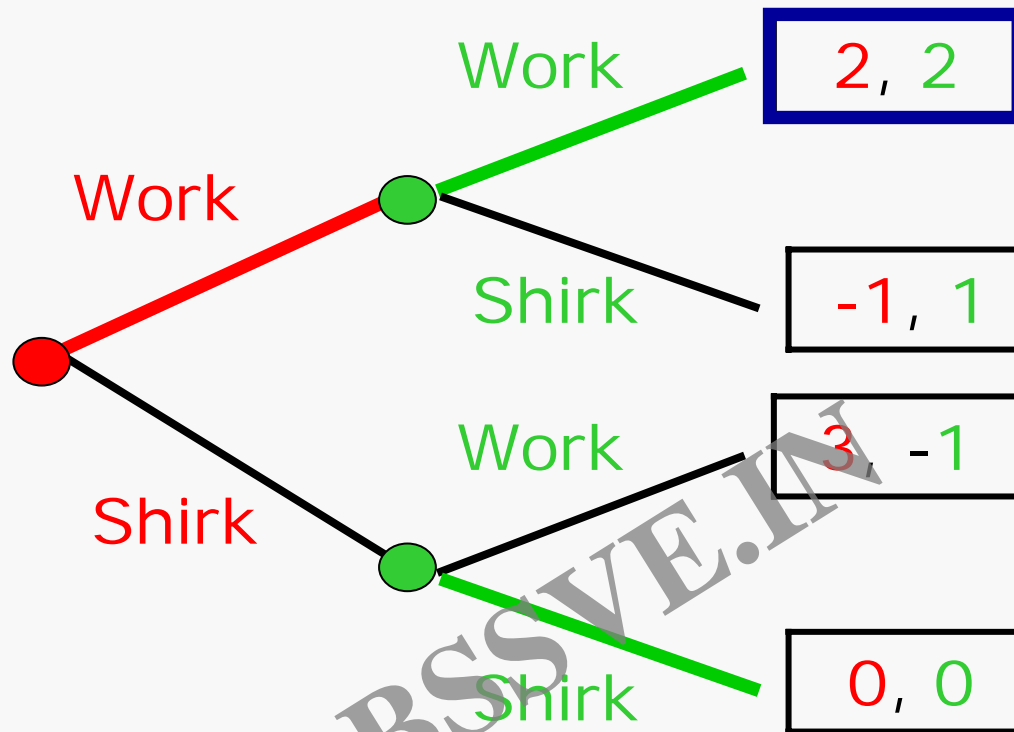
The table is annotated with arrows: a green arrow points left from the (2, 2) cell, a red arrow points down from the (2, 2) cell, a red arrow points down from the (-1, 1) cell, and a green arrow points right from the (3, -1) cell. A blue box highlights the (0, 0) cell.

- What should Husband do?

Lazy Husband Game



Not Dominant Anymore ...



- Husband commits *not* to Shirk

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Strategic Moves

*"The Power to Constrain
an Adversary Depends Upon
the Power to Bind Oneself."*

- Thomas Schelling









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“What’s For Dinner?”

- Child decides whether to eat veggies
 - wants to not eat veggies
 - wants very much to eat dessert

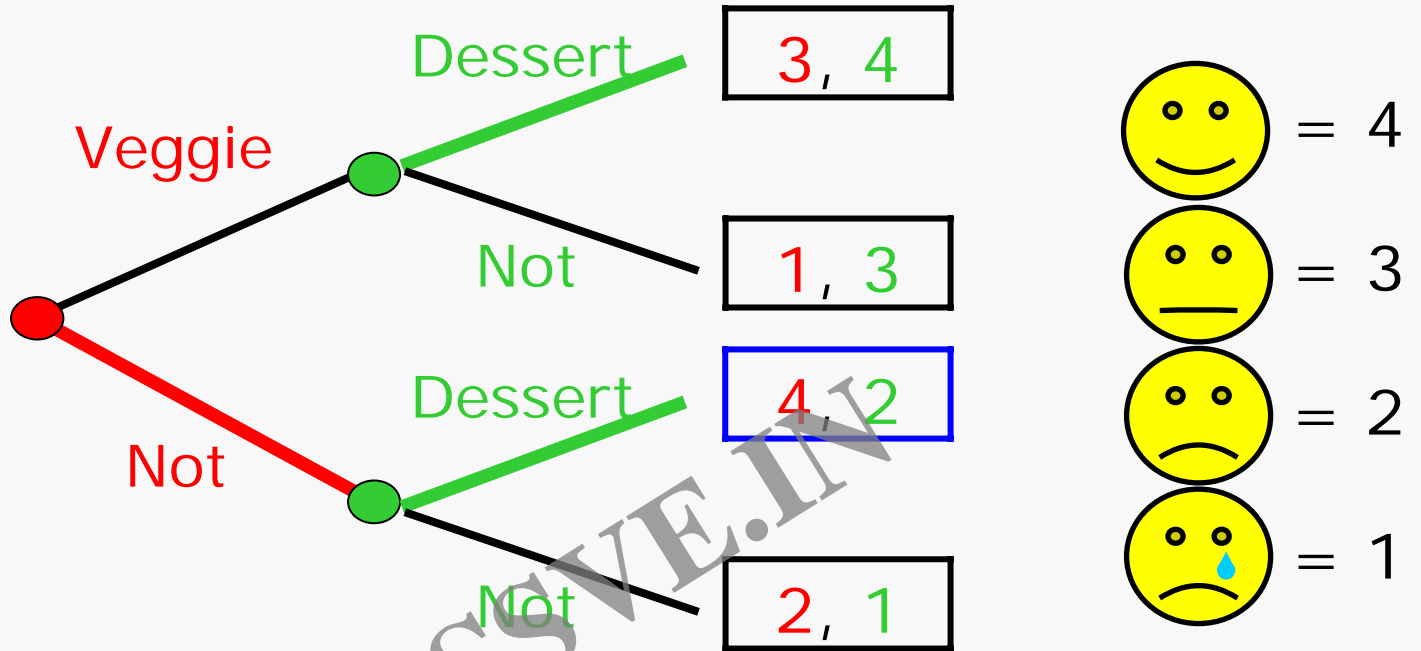
- Parent decides if dessert will be served
 - wants Child to be happy
 - wants very much for Child to eat veggies

Payoffs in Dinner Game

		Parent	
		Dessert	Not
Child	Veggies	 	 
	Not	 	 

- In simultaneous move version, both have dominant strategies: No Veggies and Dessert

Dinner w/ Sequential Moves



- No Veggies & Dessert is unique subgame-perfect equilibrium when Child moves first

Strategic Moves at Dinner

- Does moving first help Parent?
- What should Parent do?

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“You don’t get dessert *unless* you eat veggies”

- Parent can improve outcome by committing to a *response rule*
- “Threat” = response rule that punishes others if they do not cooperate with you, in a way that goes *against* your own incentives
- “Promise” = response rule that rewards others if they do cooperate, in a way that goes *against* your own incentives

“You don’t get dessert *unless* you eat veggies”

- This is a “threat” in our game
 - Parent will act against incentive to give Dessert if Child doesn’t eat veggies
- Would be a “promise” if Parent would normally not give Dessert *even if* Child ate veggies

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Threats and Promises

- A school bully says to me: “I will beat you up if you come to school”
 - This is a fact, not a “threat”!!
 - Saying this will not change whether I decide to go to school
- To be effective, promised or threatened actions must go *against* one’s own incentives

Students' Dilemma

- Two students in a class. Professor announces an unusual "final exam":
 1. *"If neither of you shows up to class tomorrow, you both get A's.*
 2. *"If both of you show up, you both get B's.*
 3. *"If one of you shows up, that person gets an A plus TAship for next year, while the other person gets an F."*
 - *both students would like the TAship*
- How can the students escape this Prisoners' Dilemma?

Escaping the Prisoners' Dilemma

*"I won't confess if you don't
but I will confess if you do"*

- Is this a promise or a threat?
- If credible, what is its effect?

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Warnings and Assurances

- Just stating what you will do without commitment is called a “warning” or “assurance”
- Warnings and assurances are effective for coordinating behavior when there are multiple Nash equilibria
 - To be effective, warnings or assurances must be *consistent with* one’s own incentives

Talk is Cheap

“Continental Airlines said yesterday that it would raise airfares on about two-thirds of its routes ... to take effect September 5.”

- Reuters. “Continental Raising Fares,” *The New York Times*, 29 August 1992.

“Continental Airlines has dropped its plan to raise domestic airfares by 5%.”

- Carroll, Doug. “Airlines Delay Fare Increases,” *USA Today*, 4 September 1992.

Talk is Cheap ...

Boeing Co. "announced it was building a plane with 600 to 800 seats, the biggest and most expensive airliner ever. Some in the industry suggest Boeing's move is a bluff to preempt Airbus from going ahead with a similar plane."

- Rothman, Andrea, and Dori Jones. "Boeing Launches a Stealth Attack on Airbus," *Business Week*, 18 January 1993.

... And Getting Cheaper

Airbus announces commercial launch of the A3XX, the largest civil aircraft ever.

"Boeing ... has said that there is no market for such a large plane and has decided to modernize its trustworthy 747 family of planes rather than build its own megaseater."

- Seward, Deborah. "Airbus Announces Launch of A3XX Superjumbo Jet." *Associated Press*, 23 June 2000.

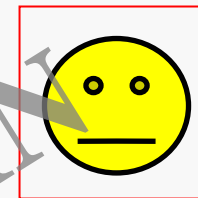
Reagan Tax Plan

Repubs

Support Reagan Completely

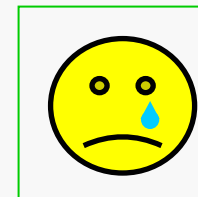
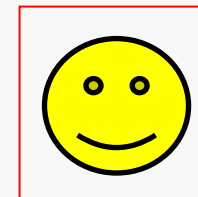
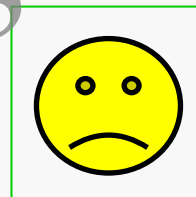
Compromise

Mostly Support Reagan



Dems

Attack Reagan



Senate vs. House

- Senate Dems hoped for compromise by not attacking Reagan ... but Repubs didn't yield
- Can House Dems do anything to get a better outcome?

In-Class Game Next Time

- Please prepare for “Dynamic Pricing Game” to be played next class
 - See handout

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Lecture 6 Game Plan

- Strategic moves continued
 - ... how to be credible
- Dynamic Pricing Game
- Strategic substitutes and complements
 - commitments to be tough vs. soft
 - puppy dog ploy, lean & hungry look, etc.

Trucking Entry (from Chevalier reading)

- A new trucking company is considering entering two *natural monopoly* markets for hauling agricultural products
 1. Market 1 currently served by railroad with $MC = \$.20/\text{ton-mile}$ and track that cost \$20 million
 2. Market 2 currently served by trucking company with $MC = \$.20/\text{ton-mile}$ and trucks that cost \$20 million

Irreversibility

- Sunk cost of track is irreversible for the railroad
 - railroad has credibly committed to stay in this market, regardless of whether another firm comes in
- Cost of trucks is fixed cost, not sunk
 - truck company can not credibly commit to stay in the market after entry

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Conventional Wisdom

- Don't burn bridges.
- Decrease downside risk.
- It is nice to have more options.
- *This ignores the strategic value of commitment!*

Strategic Commitment

- You are not credible if you propose to take actions that go against your own incentives ...
- How to be credible?

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Credibility

■ Remove strategies

- from your own set of future choices the strategies that may tempt you in the future
- Example: giving away your patent

■ Reduce payoffs

- from those strategies that may tempt you
- Example: customers as hostages.

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Removing Strategies I

■ Delegation

- In contract negotiation, can “squabble” over many details
- Instead, send an agent with power of attorney to “sign as is” or “walk away”
- Haggling over prices in a department store

Learn from government bureaucracy:

“The rules won’t allow me to do what you ask”

Removing Strategies II

- Burning Bridges
 - Power comes from *not* being able to retreat

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Burning Bridges

*"A surrounded enemy must
be given a way out"*

- Sun Tzu in "Art of War", 400BC

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Burning Bridges: Example 1

■ Semiconductor patent sharing

“Mosaid Technologies, a designer and licensor of semiconductor chips and technologies, just announced a patent sharing deal with Mitsubishi Electric”

- Share patent with another competing firm
- Commit to chip supply to production plants
- Commit to no opportunistic behavior

Burning Bridges: Example 2

- Polaroid instant photography
 - Refused to diversify out of its core business. With all its chips in instant photography, it was committed to fight against any intruder in the market.
 - On April 20, 1976, after twenty-eight years of a Polaroid monopoly on the instant photography market, Eastman Kodak entered the fray.

Burning Bridges: Example 2

- 12 October 1990: Court awards Polaroid a \$900 million judgment against Kodak. Kodak forced to withdraw from market

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Contracts with Third Parties

- Reducing one's own payoffs in a credible, *irreversible* way can be difficult.
- Third-parties can be useful as "enforcers"

The Bocchicchio Family

- Mafia in Italy, peaceful in the U.S.
- Mob bosses need to be able to meet safely, but no one trusts anybody
- Enter the Bocchicchio family
 - certain revenge if any Bocchicchio harmed
 - act as “hostages” for both sides.
 - if Don Corleone is killed by Don Barzini, then the Corleone family will kill their Bocchicchio
 - But then the Bocchicchio will blame *Barzini* since he promised the Bocchicchio would be safe!!

Committing to Fight

- Make yielding market share intolerable
 - adopt high FC, low MC technology
 - sign irrevocable agreements for expensive raw materials
 - load up on debt, or covenant debt so management loses control of the firm if market share slips

Reducing Your Payoffs Using Third-Parties

- Contracting with customers to commit to competitors
 - Most Favored Customer clauses

- Contracting with lenders to commit to a take-over price
 - Interest-rate rise if loan amount increases

Island Bars

- Two firms: **Firm 1** and Firm 2
- Two prices: low (\$4) or high (\$5)
- 3000 captive consumers per firm
- 4000 floating go to firm with lowest price

		Firm 2	
		Low	High
Firm 1	Low	20 , 20	28 , 15
	High	15 , 28	25 , 25

Contracting with Customers

- The game is a prisoner's dilemma
 - Both firms prefer: { High, High }
 - Only equilibrium: { Low , Low }
 - Cannot credibly promise to play High
 - Even if committed to High, other firm would still respond with Low
- How to resolve this?
 - Third-party contracts with customers

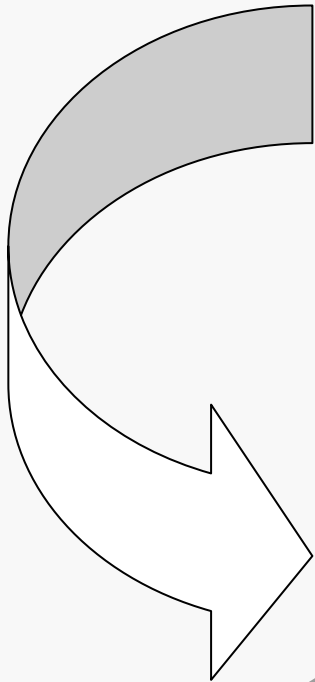
Most Favored Customer

- Say in period 1, the firms colluded and each sold to 5000 customers
- In period 2, firms must refund to last period's customers \$1 each if price is *low*
- What is the impact on the game?

Escaping the Prisoners' Dilemma with MFCs

		Firm 2	
		Low	High
Firm 1	Low	20 , 20	28 , 15
	High	15 , 28	25 , 25

		Firm 2	
		Low	High
Firm 1	Low	15 , 15	23 , 15
	High	15 , 23	25 , 25



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Incentives for MFCs

- Firms can escape Prisoners' Dilemma if they adopt MFC clauses
- BUT each firm appears to have a dominant strategy *not* to adopt
 - If other has MFC, you get 25 with MFC and 28 without. (In latter case, you price Low while other prices High)
 - If other does not have MFC, you get 15 with MFC and 20 without

Meta-Prisoners' Dilemma?

- We appear to have simply created a new Prisoners' Dilemma
- Is there any reason this Dilemma might be easier to resolve?

		Firm 2	
		MFC	Not
Firm 1	MFC	25 , 25	28 , 15
	Not	15 , 28	20 , 20

Contracting with Lenders

- Takeover offer: \$200 million
- You can “afford” \$20 million / year
- Finance takeover for 20 years at 7%
- Add penalty: if amount greater than \$200 million, +1.5 points on interest rate
- Annual Payments:
 - \$200 million: \$18.6 million / year
 - \$210 million: \$19.6 million / year
 - with penalty: \$21.9 million / year

Summary of Commitment Methods

- Reduce available strategies
- Reduce payoffs

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The Flip Side

- You want to make it difficult for opponent to commit to actions that hurt you / help opponent commit to actions in your favor
- Increase opponent's strategy space
 - Exclude bargaining agents
- Lower opponent's payoffs
 - Poison pills
- Raise opponent's payoffs
 - Reputation bolstering

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In-Class Game

Dynamic Pricing Game

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Dynamic Pricing: Rules

- Two firms. 100 customers. Zero costs.
- Stage 1: Firms decide whether to invest in creating “loyal” customers
 - “loyal” customer will always buy from you no matter what the price
 - choice: 0 loyal or 30 loyal at cost \$250
- Stage 2: Firms alternate with (non-increasing) price announcements until they stop changing. Prices start at \$50 each.
 - choices: \$50, \$40, \$30, \$20, \$10

Dynamic Pricing: Payoffs

- $\text{Payoff} = \text{Revenue} - \text{Loyalty Cost}$
- If prices stay \$50 for both firms, each firm gets Revenue = \$2500
- Otherwise they won't split market equally:
 - "bigger firm" is one that has (1) lower price or (2) was first to announce final price (if equal)
 - bigger firm sells 100 or 70 at its own price, depending on loyalty of others' customers
 - smaller firm sells 0 or 30 at its own price

Play Dynamic Pricing Game!

You will play as a team. (Pair up with another team.)

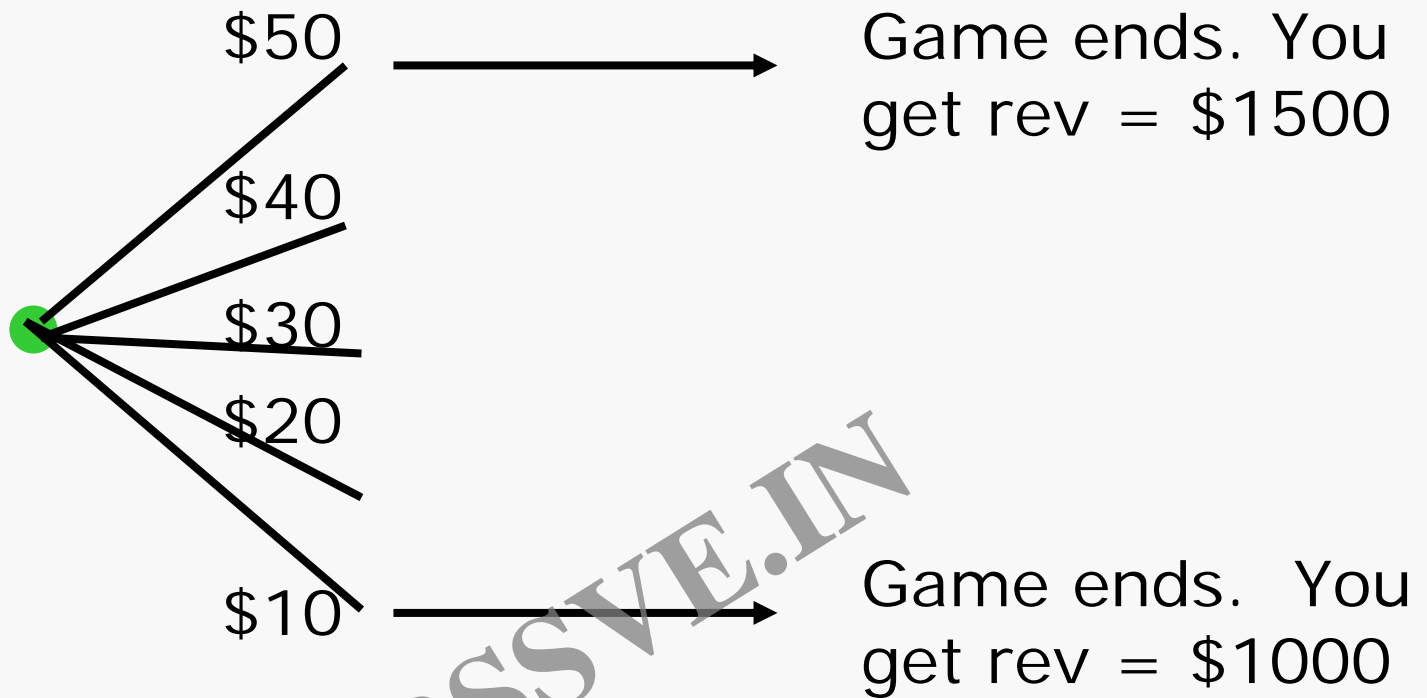
1. Play Loyalty Stage: simultaneously choose "Loyal" or "Not Loyal"
2. TA will tell you who goes first in Pricing Stage

Record game progress on handout and give this to TA at end of game

Upside of Loyalty

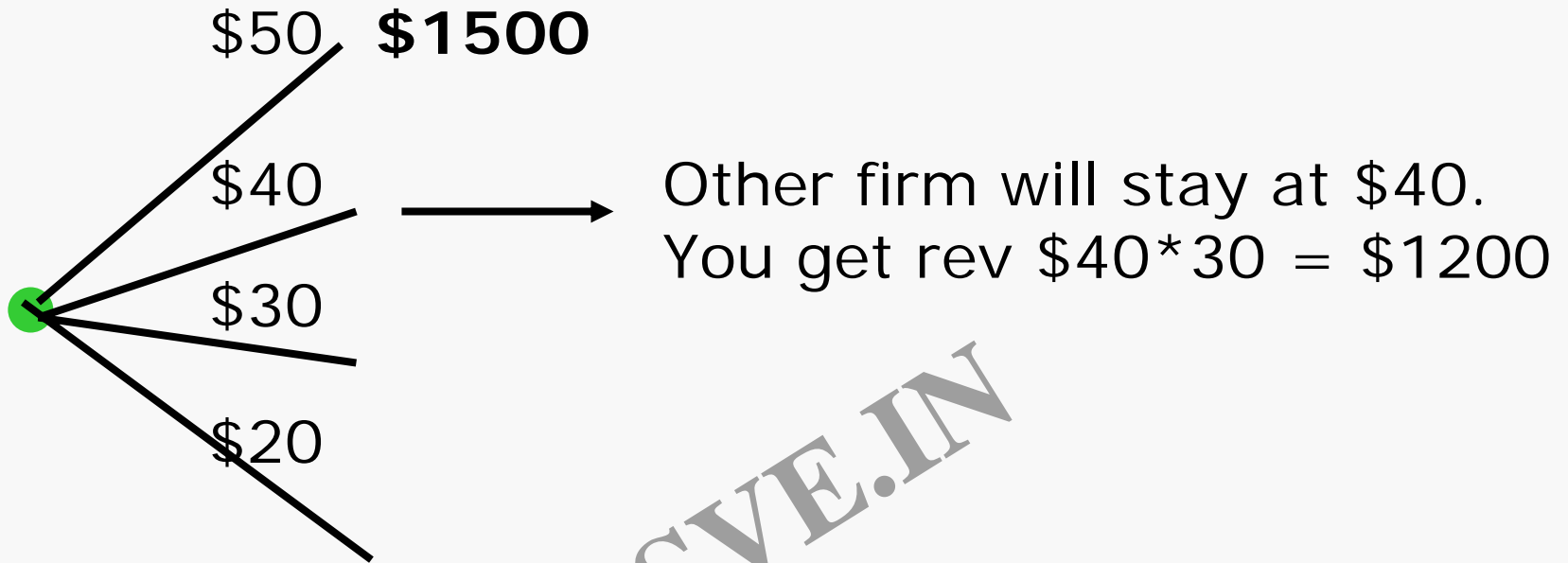
- If other firm undercuts you, your final payoff will be higher than if you had disloyal customers
- Suppose first that other has no loyal customers and undercuts with \$40
 - What would you do?

Loyal undercut by Disloyal



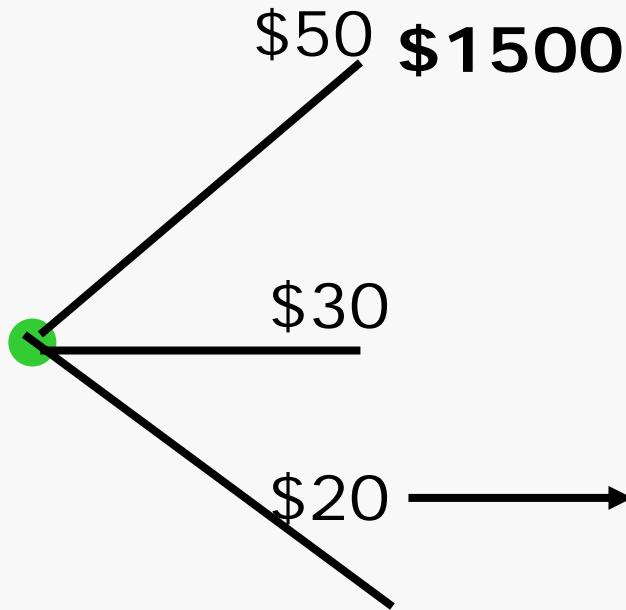
- Don't respond with \$10

Loyal undercut by Disloyal



- Don't respond with \$40
- *What about \$30 or \$20?*

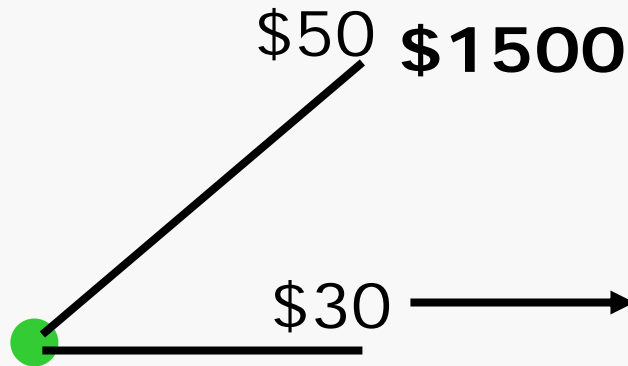
Loyal undercut by Disloyal



Since other firm has no loyal customers, it will re-undercut with \$10. You get $\$20 \times 30$

- Other firm will re-undercut for sure since it has no loyal customers

Loyal undercut by Disloyal

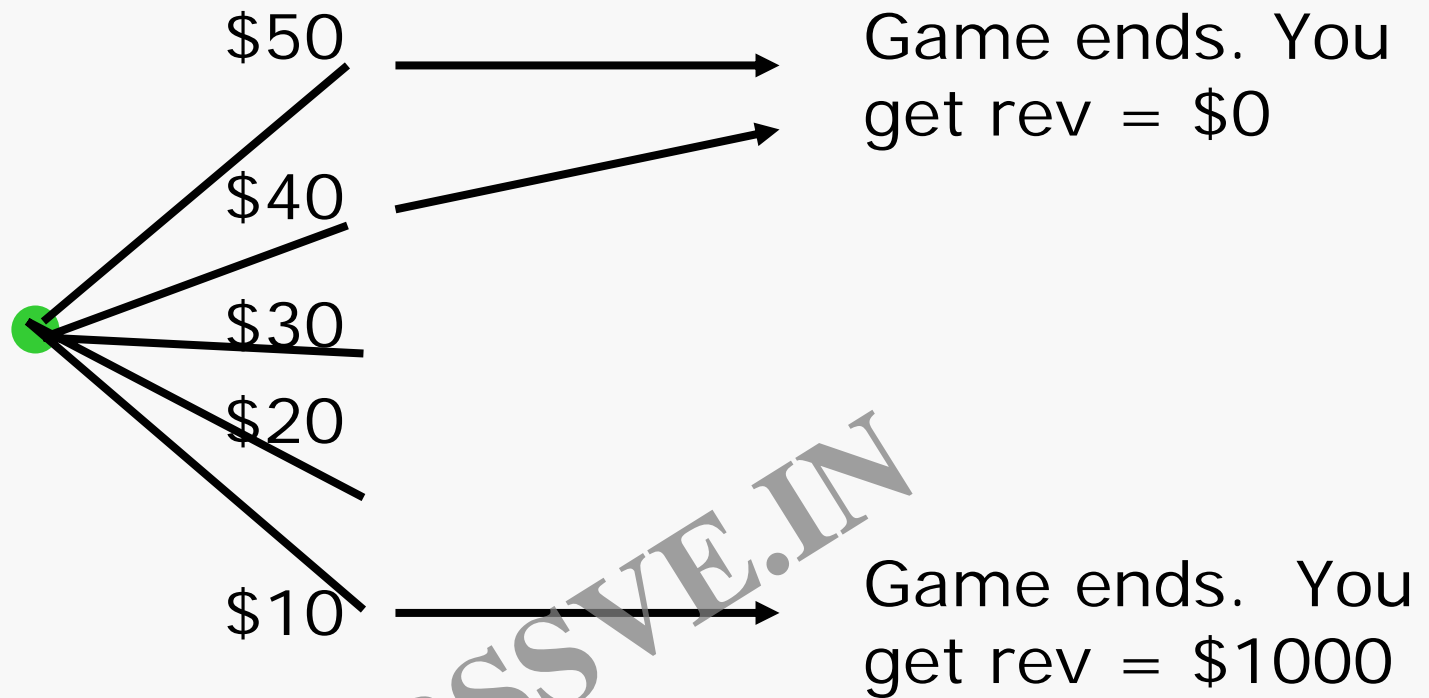


If other firm responds with \$20, you will re-undercut with \$10 since you prefer $100 * \$10$ over $30 * \$30$.

So, other firm will undercut with \$10 ending the game and you with $\$30 * 30$

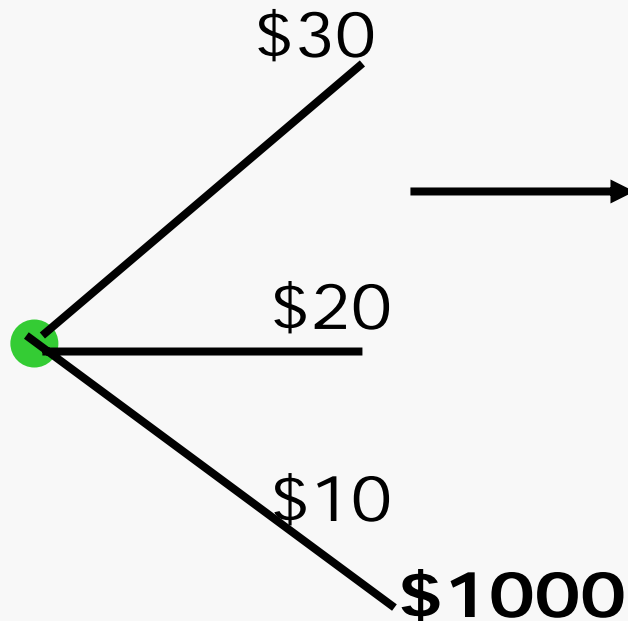
- Payoff if you have loyal customers and get undercut is $\$1500 - \$250 = \mathbf{\$1250}$

Disloyal undercut by Disloyal



- Don't respond with \$50 or \$40

Disloyal undercut by Disloyal



Since each firm has no loyal customers, each will re-undercut until price equals \$10. If you bid \$30 or \$20, other will go to \$10 and you will get zero revenue

- Respond with \$10 and get \$1000
- Not as good as \$1500 if you had Loyal

Downside of Loyalty

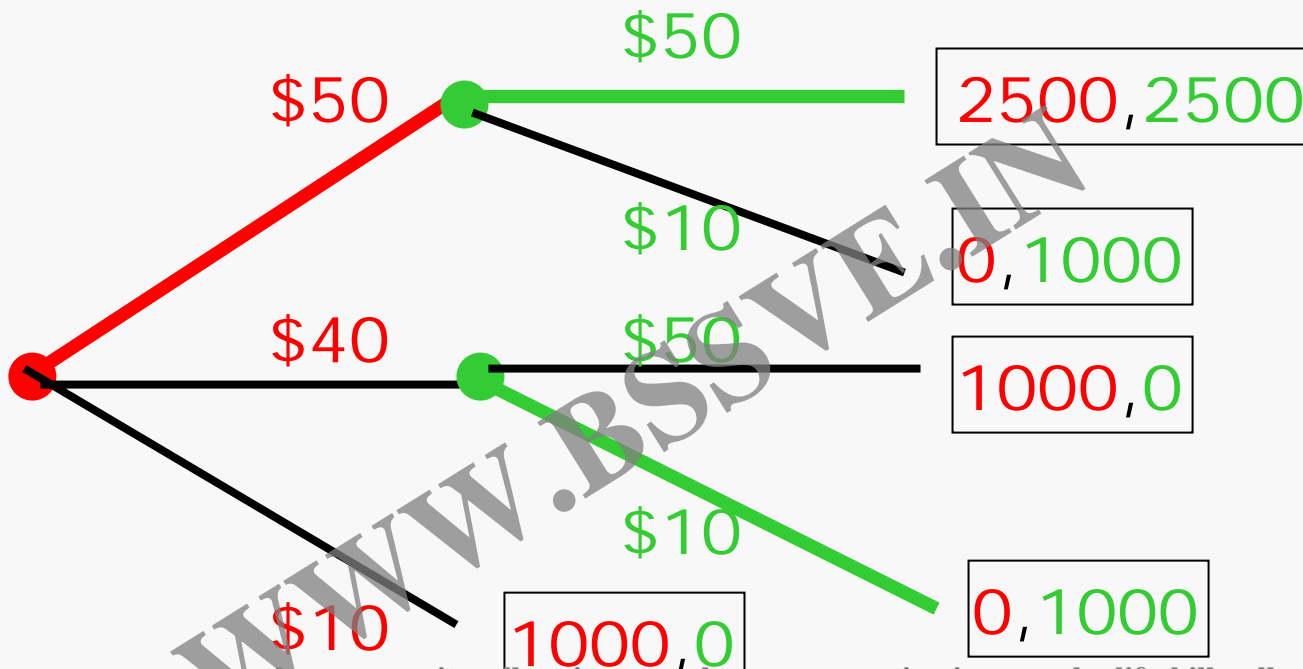
- Your unwillingness to re-undercut makes you an easy target
 - Disloyal opponent (whether first or second) will undercut you with \$40, leaving you with only \$1500
 - Loyal opponent (if first) will also undercut you with \$40
 - this is not obvious but can be shown

Upside of Disloyalty

- You are so “Lean & Hungry” that no Loyal opponent messes with you
- Against Loyal opponent, you get $70 * \$40 = \2800
- What about against Disloyal?

Disloyal vs. Disloyal

- Any undercutting must lead to ultimate price of \$10 → no better than \$1000
 - no price war in subgame-perfect equilibrium!



To Be Loyal or Not To Be ...

	Loyal	Disloyal
Loyal	<p><i>Chicken</i></p> <ul style="list-style-type: none">■ 1st gets \$2800■ 2nd gets \$1500■ Average \$2150	<p><i>Loyal Servant</i></p> <ul style="list-style-type: none">■ DL gets \$2800■ L gets \$1500
Disloyal	<p><i>Loyal Servant</i></p> <ul style="list-style-type: none">■ DL gets \$2800■ L gets \$1500	<p><i>Assurance</i></p> <ul style="list-style-type: none">■ Both get \$2500

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Commitments to be Tough

“At the critical moment, the leader of an army acts like one who has climbed up a height, and then kicks away the ladder behind him”

- Sun Tzu, “The Art of War”, 400 BC

Commitments to be Soft

“What is more fluid, more yielding than water? Yet back it comes again, wearing down the rigid strength that can not yield to withstand it. So it is that the strong are overcome by the weak, the haughty by the humble.”

- Lao Tzu, “Tao Te Ching”, 600 BC

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Club Z

- Zeller's is a major Canadian mass-merchandiser
- 1/3 of Canadians belong to Club Z, Zeller's "loyalty rewards" program
- 4/5 of Zeller's shoppers are in Club Z
- Club Z appears to give Zeller's an advantage over competitors. At any given price, more people will buy from Zeller's

Potential New Entry

- In 1990, Q-Co is considering entering the Canadian market.
- Would Club Z give Zeller's an advantage in deterring entry?

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Competing with Wal-Mart

- In 1990s, Wal-Mart is (definitely) entering the Canadian market.
- Does Club Z give Zeller's an advantage in price competition with Wal-Mart?

Tough or Soft?

- Three main factors determine whether to be tough or soft:
 1. Does your opponent view strategies as strategic substitutes or complements?
 2. Do you want your opponent to be more or less aggressive?
 3. Are you trying to induce exit/deter entry or to deal with an entrenched opponent?

Strategic Substitutes vs Strategic Complements

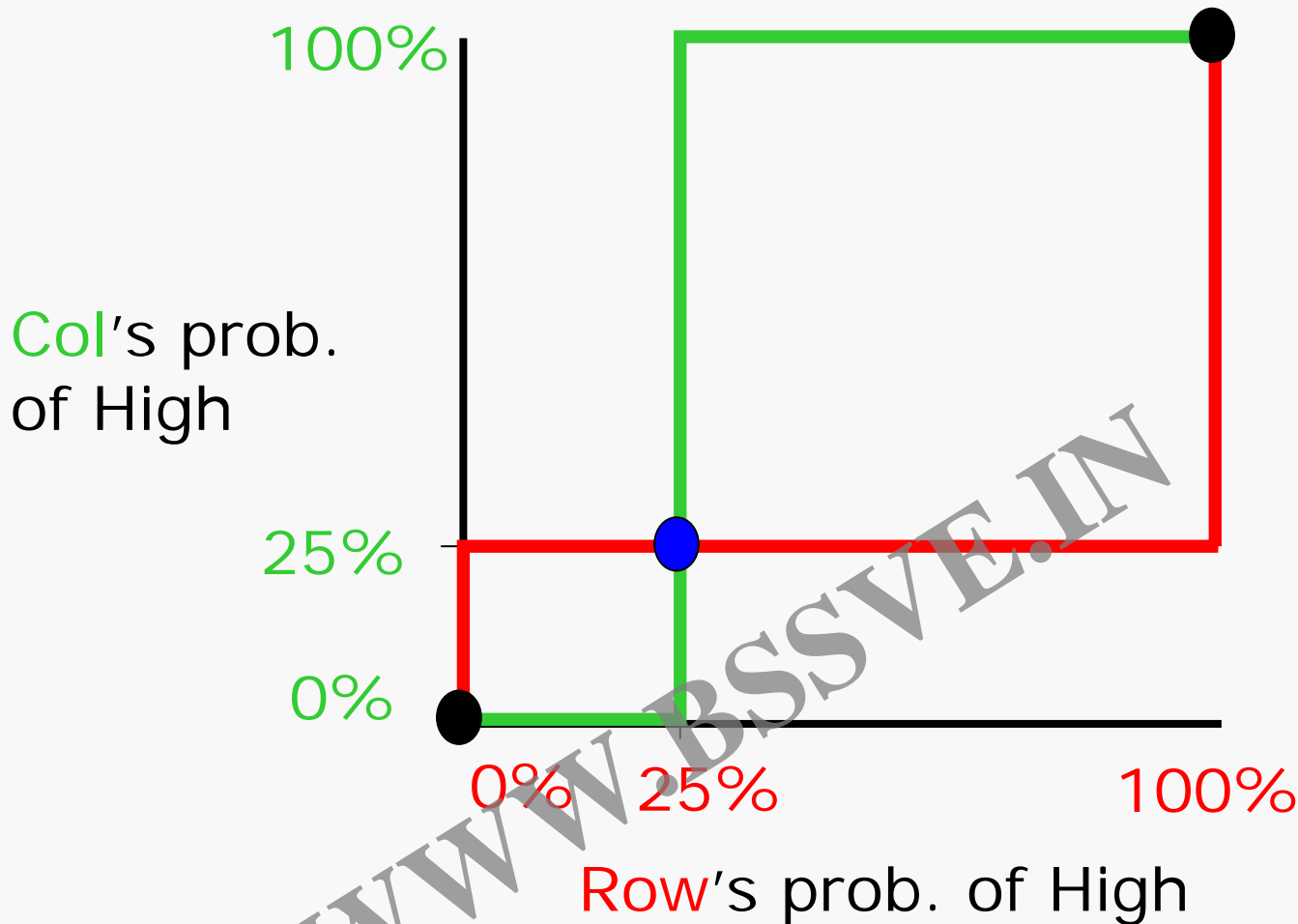
- Player A views the strategies as **strategic substitutes** if its reaction curve is downward-sloping
 - i.e. A prefers to be less aggressive if other player is more aggressive
- Player A views the strategies as **strategic complements** if its reaction curve is upward-sloping
 - i.e. A prefers to be more aggressive if other player is more aggressive

Strategic Substitutes vs Strategic Complements?

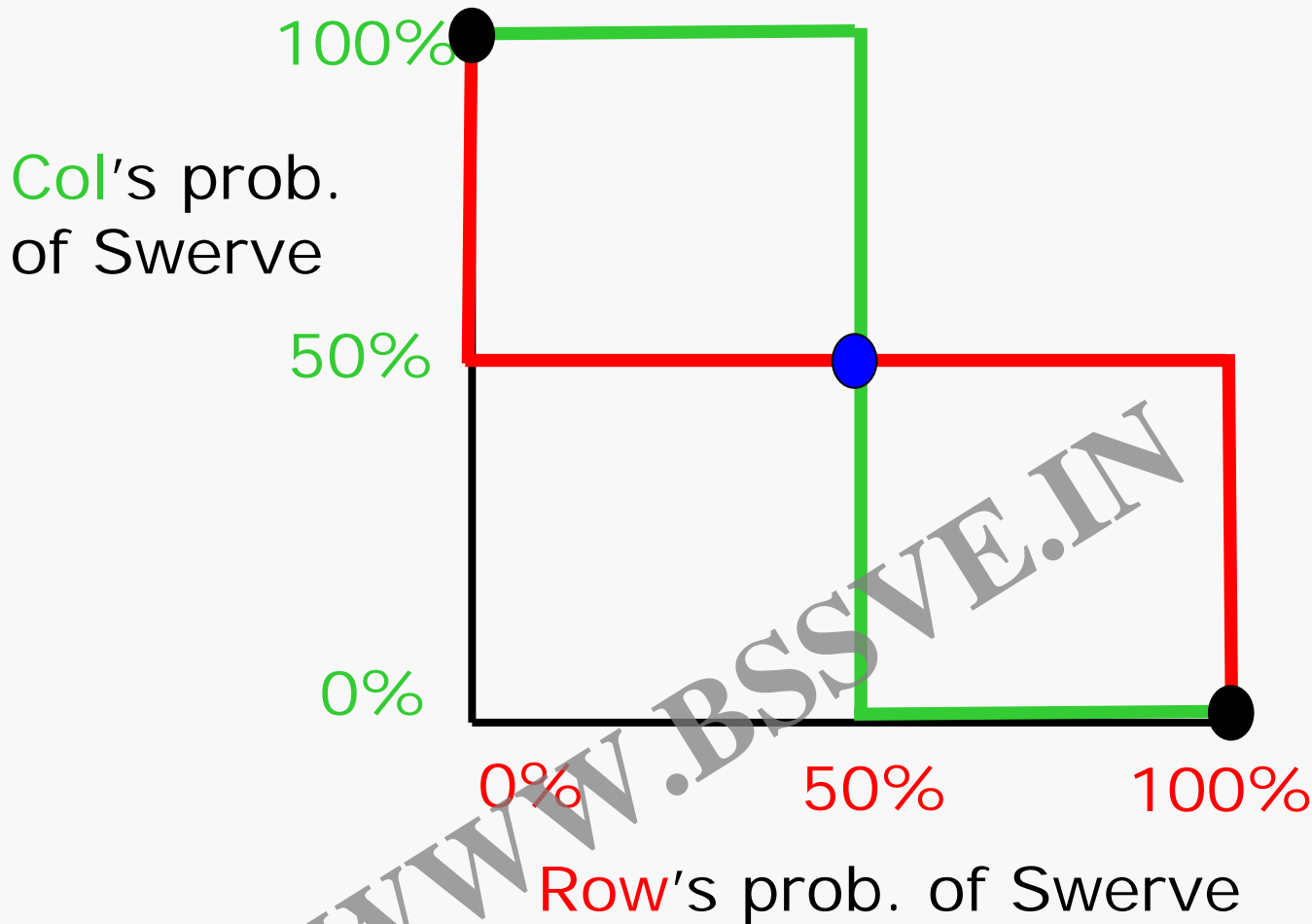
- Which is in strategic substitutes and which in strategic complements?
 - competing on price
 - competing on capacity/quantity
 - competing on advertising
 - competing on research

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Assurance Game: Strategic Complements



Chicken Game: Strategic Substitutes



Competition vs Reliance

- Do you want the other player to be more or less aggressive?
 - Interpretation of “more aggressive” depends on the situation and need not have anything to do with actual aggression
- **“Competition”**: any game in which players want others to be less aggressive
- **“Reliance”**: any game in which players want others to be more aggressive

Competition with Investment

- Firms often have the opportunity to take an action prior to a game that makes it tend to be more or less aggressive than otherwise
 - Such actions are called **“investment”** though they may actually not be related to any actual investment

A Lot or a Little Investment?

- You want to commit to do more of whatever will lead the other player to be less aggressive
- So, decision to invest a lot or a little depends only on two factors:
 1. Does your investment lead you to be more or less aggressive?
 2. Does your opponent view strategies as strategic substitutes or complements?

Taxonomy of Optimal Competitive Strategies

More Makes You Tougher

More Makes You Softer

Strategic Complements

Puppy Dog

less investment makes you softer, makes other less aggressive

Fat Cat

more investment makes you softer, makes other less aggressive

Strategic Substitutes

Top Dog

more investment makes you tougher, makes other less aggressive

Lean & Hungry

less investment makes you tougher, makes other less aggressive

Puppy Dog: Serve a Niche

- Suppose firm is developing a product to compete with that of dominant firm
- “Investment” = potential clientele
 - Niche product won't attract customers from dominant firm, decreasing your incentive to compete on price
 - This makes dominant firm less aggressive since prices are strategic complements

Top Dog: Export Subsidies

- Suppose domestic firm competes in quantities in a foreign market
- “Investment” = export subsidy
 - With export subsidy, domestic firm will export more into foreign market
 - This leads foreign firm to produce less

Summary

- The ideas of strategic substitutes and complements organize many strategic intuitions in a systematic framework
 - How best to play against an entrenched opponent (including “Judo Economics”)
- Next time:
 - How best to induce exit / deter entry
 - Application to an entry game (Ryanair)

Case for Next Time

- Prepare “Ryanair” Case for discussion in next class.
 - See handout

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Lecture 7 Game Plan

- More on strategic substitutes and complements
- Application to Entry: Ryan Air
- Strategic exercise of real options

Systems and Product Compatibility

- Tape deck + speaker = sound *system*
- Two firms produce both tape decks and speakers.
- First, firms decide whether or not to allow for compatibility
 - products are compatible if both allow for it
- Second, firms compete on price

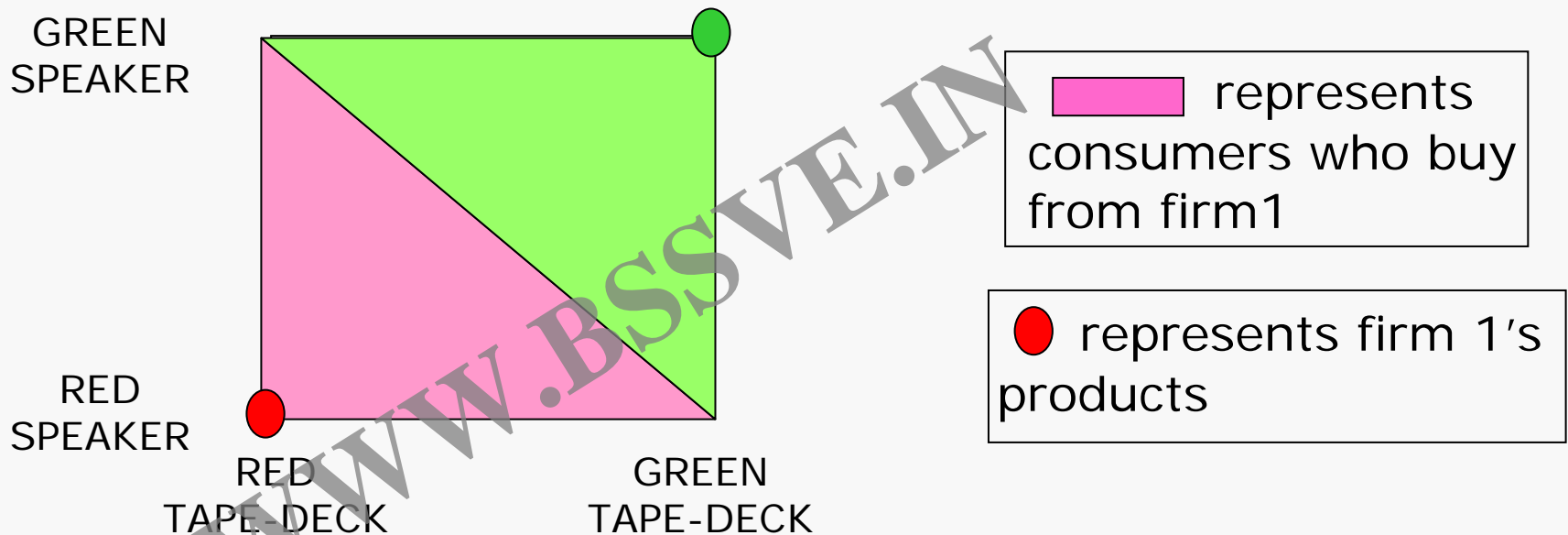
Sound System Consumers

- Each firm's products are described as corners of a square.
- Consumers are represented as points in the square
- Each consumer buys from whatever firm has lowest *price + distance*
 - "*distance*" captures comparative preference for the products


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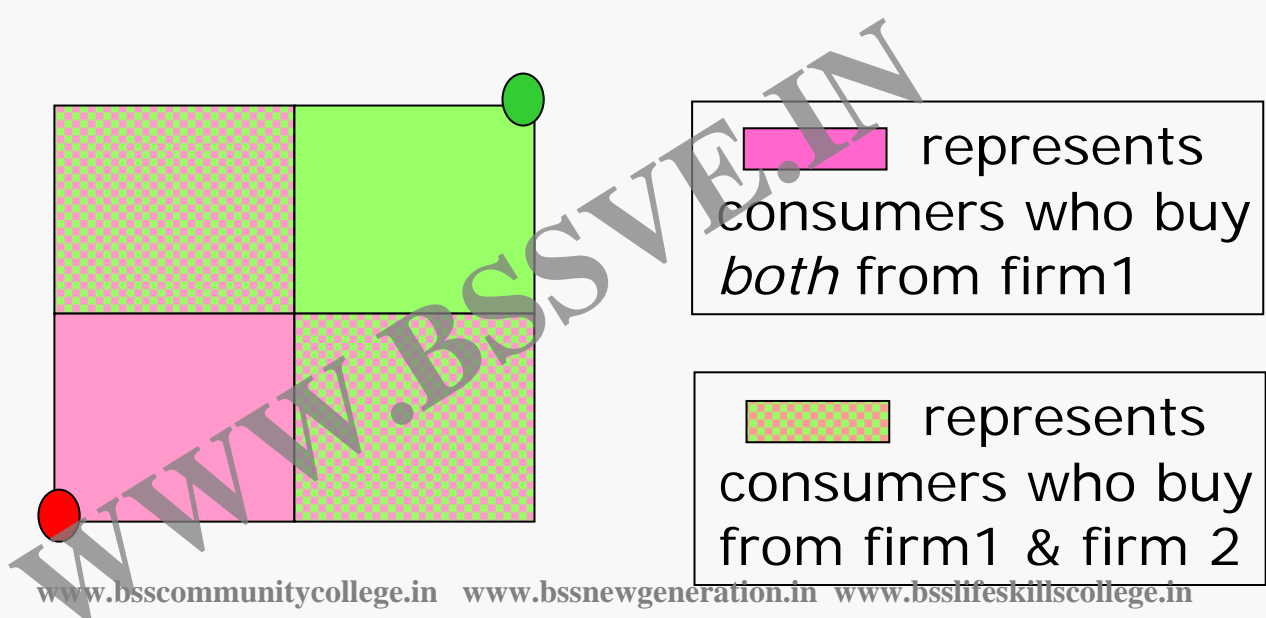
Incompatible Competition

- Consumers in upper-left and lower-right prefer to mix and match



Compatible Competition

- Consumers are better matched in  regions
- AND price competition is less intense



Compatible Competition Less Intense

- When firm 1 lowers price of RED SPEAKER, it *raises* demand for GREEN TAPE-DECK
 - tends to lead GREEN to *raise* its price
- Under incompatibility, lowering price of RED SPEAKER *lowers* GREEN TAPE-DECK demand
- The firms appear to have a **common interest in compatibility**

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Incompatibility & Exit

- Previous discussion implicitly assumed entrenched firms
- If firm 1 wants to drive firm 2 out of the market (or deter entry), compatibility is a bad approach since it increases both firms' payoffs

Strategies to Induce Exit in Competitive Games

More Makes You Tougher

More Makes You Softer

Strategic Complements

Top Dog

more investment makes you more aggressive, hurting other player

Lean & Hungry

less investment makes you more aggressive, hurting other player

Strategic Substitutes

Top Dog

more investment makes you more aggressive, hurting other player

Lean & Hungry

less investment makes you more aggressive, hurting other player

Reliance Games

- Conclusions for competitive games apply to reliance games, except “flipped”
 - to induce *entry*, do whatever makes you more aggressive

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Entry

*“The wise win before they fight,
while the ignorant fight to win.”*

-Zhuge Liang, chief military strategist,
Shu Kingdom, 200 AD

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Real Options

- Entering a new market
 - Option to enter later
- Investments in large, risky projects
 - Option to delay and to stop
- New technologies: multi-stage option
 - R&D → patent → testing → pilot →

Parking Lots

- You own an unprofitable undeveloped parking lot downtown
- Additional parking revenues would be more than sufficient to cover costs of building a multi-level parking garage
- Building a garage is a no-brainer, right?

A Tale of Two Parking Lots

- Another undeveloped lot sits right next to yours
- Does the presence of this other lot make you more or less eager to build an office building on your land?

Strategic Incentives to Exercise Real Options

- First-mover effect that we've seen before
→ Want to move first exactly when:
 - investments are strategic substitutes OR
 - investments are strategic complements and reaction curves have slope > 1
- Informational benefit of observing others' investment → Want to move last
 - *others' decision* to invest is "good news" about their information
 - first-mover suffers *Winner's Curse*, see Lecture 9
 - *outcome* of investment also informative

Exercising Cascades

- When others exercise an option this reveals *positive* information
- Furthermore:
 - more *information* → less option value → others more likely to exercise
- This can lead to cascades in which many firms follow an early exerciser
 - Example: "Miracle on Sixth Avenue"

Summary

- When considering entering a market
 - anticipate and control incumbents' incentive to retaliate
 - consider how your entry may change followers' decision whether/how to enter themselves

In-Class Game Next Time

- Please prepare for “Angry Negotiation” to be played next class
 - See handout

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Lecture 8 Game Plan

- Retaliation, escalation, and disarmament

- Brinkmanship
 - Angry Negotiation Game

- Games with hidden information

Commitment in “Dr. Strangelove”

- Severity
 - Create fear in the mind of the enemy
- Irreversibility
 - Must be irreversible
- Irrationality
 - Not something a sane man would do
- Practicality
 - Punishment shouldn't be too harsh
- Clarity
 - “Tell the world”

Surprise Attack

“There is a difference between a balance of terror in which either side can obliterate the other and one in which both sides can do it no matter who strikes first”

- Schelling, Thomas. *Strategy of Conflict*. Harvard University Press, 1960.

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Old West Gunman Game

Steve McQueen



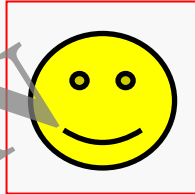


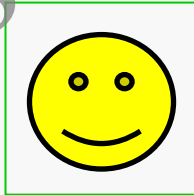


Try to Kill

Don't



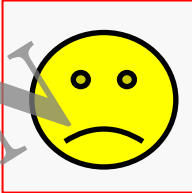


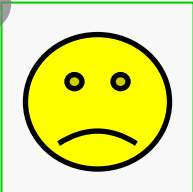


Try to Kill

Clint Eastwood

Don't

Cold War Nuclear Game

		Khrushchev			
		Preemptive Strike		Don't	
Kennedy	Preemptive Strike				
	Don't				

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Retaliation and Escalation in Business

- Price wars
- Marketing battles
- Negotiations with organized labor

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Disarmament

To escape from a game of mutual harm:

1. **stabilize**: remove your strategies that limit others' ability or *incentive to retaliate*
 - unilateral OK though simultaneous preferred
 - "retaliate" = "hurt after being hurt yourself"
2. **de-escalate**: decrease your severity of harm *while* decreasing others' severity of retaliation
 - must be simultaneous to maintain credible retaliation throughout disarmament process

Punishment Must Fit Crime

- For retaliation to be credible, you must have the ability *and incentive* to retaliate after being harmed
- USA could never credibly threaten to invade Japan over trade barriers

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Brinkmanship

*"... between one out of three
and even ..."*

- John F. Kennedy, estimating the likelihood that the Cuban Missile Crisis would lead to nuclear war, 1962

Chicken in Real Time

- Suppose you have ability to move first, but you are unsure whether your opponent will swerve
 - opponent is either "sane" or "crazy"
 - you are known to be "sane"
- What would you do?

Probabilistic Threats

- “Throw out steering wheel” has drawback that you crash when opponent is crazy
- Not doing anything also isn't good, since then your opponent will then throw out its steering wheel
- A solution is to swerve with probability in between 0% and 100%
 - must be often enough to deter “sane”
 - how might you do this, credibly?

Gradual Escalation of Risk

- Calibrating the best probability of your own craziness requires a lot of knowledge:
 - must know probability other is crazy
 - must know how much *the sane type* wants to avoid crashing
- Without this knowledge, you can still “probe” the others’ limits through a gradual escalation of the risk
 - i.e. disable steering wheel a little at a time

Conditions for Successful Brinkmanship

For this graph, see Figure 13.5 in the course textbook:

Dixit, Avinash K., and Susan Skeath. *Games of Strategy*. New York, NY: W. W. Norton & Company, 1999. ISBN: 0-393-97421-9.

How Might Kennedy Learn about Soviet Craziness?

1. (See page 457 in Dixit, Avinash K., and Susan Skeath. *Games of Strategy*. New York, NY: W. W. Norton & Company, 1999. ISBN: 0-393-97421-9.)
2. Adverse selection among those who do not yield to a given threat
 - or, in other words, not yielding may be an effective signal of craziness

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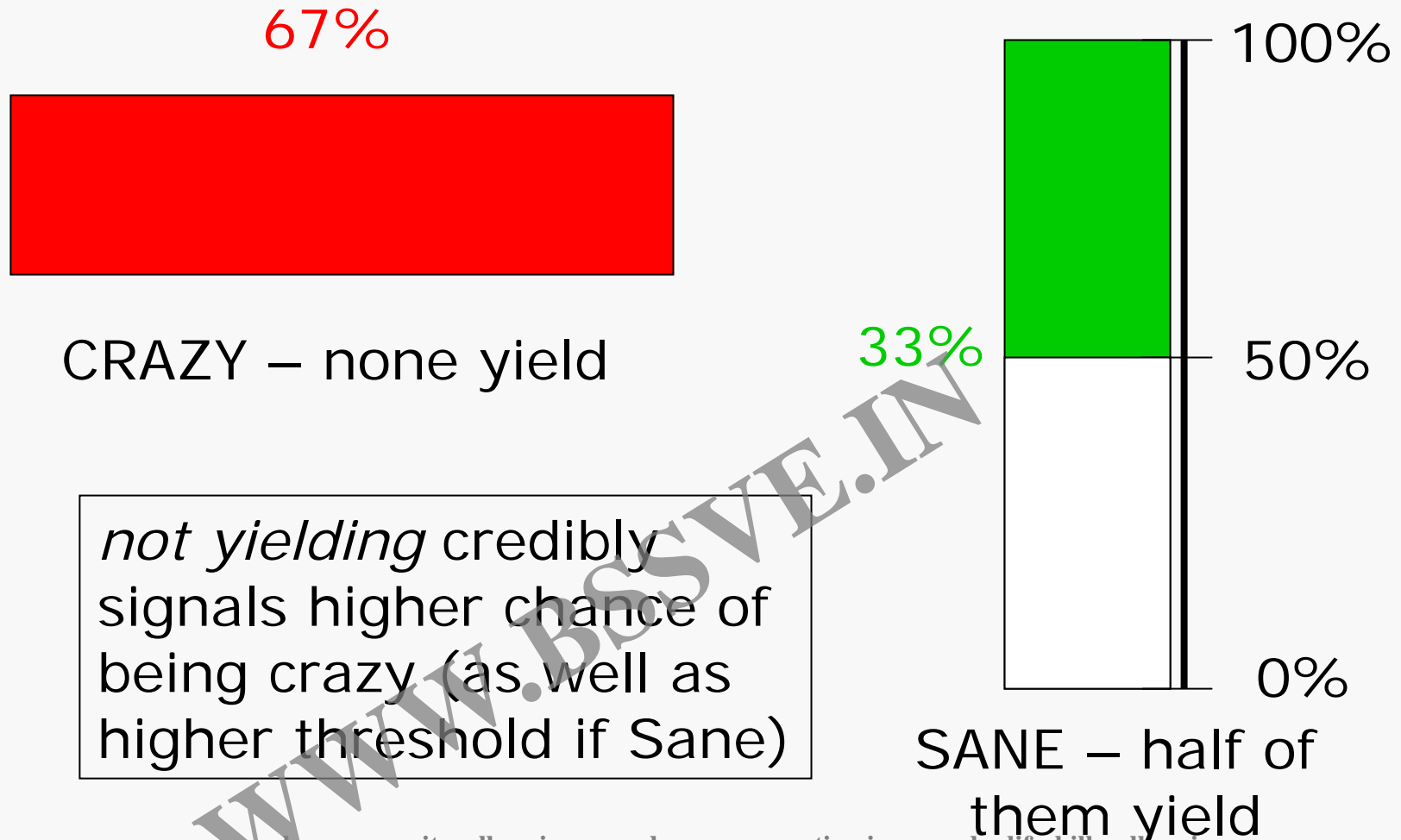
Example: Adverse Selection in Wars of Attrition

- For simplicity, suppose Kennedy believes that the Soviets are either Crazy (50%) or Sane (50%).
- Among the Sane, however, the likelihood of war needed to make them back down ranges all the way from 0% to 100% (all equally likely)

Kennedy's Initial Belief



Kennedy's Belief After Threat $q = \frac{1}{2}$ Ignored



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In-Class Game

Angry Negotiation Game

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Angry Negotiation: Rules

- Union and Management in an all-or-nothing dispute (no compromise)
- Each round, both players decide whether to Yield or Not.
 - If either Yields, the game ends
 - Otherwise, someone gets Angry with probability $10\% * (\# \text{rounds so far})$
 - if someone gets Angry, the game ends
 - if not, we continue to next round

Angry Negotiation: Payoffs

- Angry leads to payoff of 0 for everyone
- If Union yields, it gets payoff U .
If Management yields, it gets payoff M
 - U, M each either 100, 200, or 400 w/equal prob
 - if both yield at same time, both get this
- If Union yields and Management does not, Management gets $M+100$
- Vice versa, Union gets $U+100$ if ...

Get Angry!

You will play as pairs. (Choose a partner and find another pair to play against.)

We will provide your value (100, 200, or 400) and a die to roll to determine anger

Record game progress on handout and give this to TA at end of game

“Don't Yield Immediately”

- Your opponent remarks before playing:

“Even if you have the highest value for avoiding failure (400), the prospect of winning an extra 100 is worth the 10% risk of losing 400 in Round 1.
So, no one should ever yield in Round 1.”
- Is this correct?
- What would you say / do back?

“Don't Ever Yield”

- Your opponent remarks before playing:

“The way to play this game is to tell the other player that you will never Yield. That forces them to Yield (and Yield immediately) ... just so you know, I'm never going to Yield.”
- Is this correct?
- What would you say / do back?

What About Against Me?

- Suppose I am your opponent
 - someone who is *known to know* game theory inside and out
 - not necessarily an advantage!
- You are allowed to make either of these statements before the game
- I am not allowed to say anything either before or during play

(Perceived) Stupidity as Strategic Force

- If you say “I will never yield, so you must yield”, I will call your bluff
 - By not yielding, I prove that I don’t believe your threat that you will never yield. So the threat loses its teeth.

- If you say “It’s better for me not to yield in Round 1 (or Round 2!)”, I get worried
 - Perhaps you really believe this
 - I now have incentive to yield immediately
 - Conveying *mistaken beliefs* can be an effective strategy

Deception

“All warfare is based on deception”

- Sun Tzu, “The Art of War”, 500BC

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Deception

“Apparent confusion is a product of good order, apparent cowardice of courage, apparent weakness of strength”

- Sun Tzu, “The Art of War”, 500BC

Summary

- Many games involve uncertainty about other players' payoffs
- One can learn about others through their actions, in a "fooling-proof" way
 - it's too costly for other sorts to try to fool you
- Brinkmanship is one sort of example
 - those who don't give in are least afraid of disaster (or most wanting to "win")
- *Next two lectures:* More on the strategic impact of hidden information.

Online Game #8 (Takeover Bidding)

- Play Online Game #8 prior to midnight before next lecture.
- Note: We are *not* playing the games in their numerical order!!

Appendix:

Yielding in Round 1

- Someone must *sometimes* yield in Round 1 (i.e. w/ positive probability)
- Suppose not, that the first time anyone ever yields is Round $K > 1$
- But someone planning to yield in Round K would do better yielding in Round 1
 - for same reason, yielding must *sometimes* occur in every round until no one is left

Appendix:

Yield This Round or Next?

- Union type yielding in round K must prefer that to waiting until round $K+1$.
- **Benefit to yielding** is you avoid risk of anger: $U * (K * 10%) * (1 - p_K)$
- **Benefit to waiting** until round $K+1$ is that other may yield now: $100 * p_K$
 - p_K is probability that other yields in round K

Appendix:

Round 1 Equilibrium Play

	Risk to Wait	Gain to Wait	Critical % Yielding	% Higher-Value Types
400-type	40	100	$4/14 = 29\%$	0%
200-type	20	100	$2/12 = 17\%$	33%
100-type	10	100	$1/11 = 9\%$	67%

- No 200- or 100-types yield
 - if so, all 400-types must also yield
 - but 33%+ yielding means 200- and 100-types should *not* yield
- All 400-types yield
 - since $29\% < 33\%$, only 29/33 of the 400-types yield
 - if all yielded, none of them would want to yield

Appendix:

Round 2 Equilibrium Play

	Risk to Wait	Gain to Wait	Critical % Yielding	% Higher-Value Types
400-type	80	100	$8/18 = 44\%$	0%
200-type	40	100	$4/14 = 29\%$	6%
100-type	20	100	$2/12 = 17\%$	53%

- No 100-types yield
 - if so, all 200-types must also yield
 - but 53%+ yielding means 100-types should *not* yield
- All 400-types yield (6% of remaining population)
- Only *some* 200-types yield
 - since $53\% > 29\%$, 200-types would have incentive *not* to yield if they all yielded → only 23/47 of them yield

Appendix:

Round 3 Equilibrium Play

	Risk to Wait	Gain to Wait	Critical % Yielding	% Higher-Value Types
400-type	N/A	N/A	N/A	N/A
200-type	60	100	$6/16 = 38\%$	0%
100-type	30	100	$3/13 = 23\%$	34%

- All 200-types yield
 - since $34\% < 38\%$, all 200-types must yield
- No 100-types yield
 - since $34\% > 23\%$, no 100-types yield

Appendix:

Round 4 Equilibrium Play

	Risk to Wait	Gain to Wait	Critical % Yielding	% Higher-Value Types
400-type	N/A	N/A	N/A	N/A
200-type	N/A	N/A	N/A	N/A
100-type	40	100	$4/14 = 29\%$	0%

- 29% of remaining 100-types yield
 - any less and all would want to yield
 - any more and none would want to yield
- ... $5/15 = 33\%$ of remaining 100-types yield in Round 5, etc...

Lecture 9 Game Plan

- Angry Madness Tournament
- Examples of auctions
- Which auction is best?
 - Revenue Equivalence Theorem
- How to bid?
 - Winner's Curse

Yield This Round or Wait?

U	Risk of Waiting	Gain of Waiting	Critical threshold
100	\$25	\$100	$p_K = 1/5 = 20\%$
200	\$50	\$100	$p_K = 1/3 = 33\%$
400	\$100	\$100	$p_K = 1/2 = 50\%$

You want to yield this round whenever chances other yields is less than threshold

Round 1 Equilibrium Play

	Critical % Yielding	% Higher-Value Types
400-type	50%	0%
200-type	33%	33%
100-type	20%	67%

- No 200- or 100-types yield
 - if so, all 400-types must also yield
 - but 33%+ yielding means 200- and 100-types should *not* yield
- All 400-types yield
 - since $33% < 50%$, all 400-types have strict incentive to yield even when they all yield

Round 2 Equilibrium Play

	Critical % Yielding	% Higher-Value Types
400-type	N/A	N/A
200-type	33%	0%
100-type	20%	50%

- No 100-types yield
 - if so, all 200-types must also yield
 - but 50%+ yielding means 100-types should *not* yield
- Only *some* 200-types yield
 - since $50\% > 33\%$, 200-types would have incentive *not* to yield if they all yielded → only 1/2 of them yield

Round 3 Equilibrium Play

	Critical % Yielding	% Higher-Value Types
400-type	N/A	N/A
200-type	33%	0%
100-type	20%	33%

- All 200-types yield
 - If only some of the 200-types yield, then all of them want to yield
- No 100-types yield
 - since $33\% > 20\%$, 100-types don't want to yield when all 200-types yield

Round 4 Equilibrium Play

	Critical % Yielding	% Higher-Value Types
400-type	N/A	N/A
200-type	N/A	N/A
100-type	20%	0%

- 20% of remaining 100-types yield
 - any less and all would want to yield
 - any more and none would want to yield

It's Tough to be Tough

- Suppose two 100-types face off
- In Rounds 1-3
 - 57.8% get angry; no one yields
- In Rounds 4-9
 - about 16.9% more get angry
 - about 13.4% yield
 - about 10.8% don't yield & get yielded to
- Only about 1.1% reach 10th round.
- **Average payoff only about \$36.**

Signaling Toughness

- Suppose you could (credibly) reveal your toughness, i.e. that $U = 100$
- Would you want to? What would happen?

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Auctions

"A Cynic Knows the Price of Everything and the Value of Nothing"

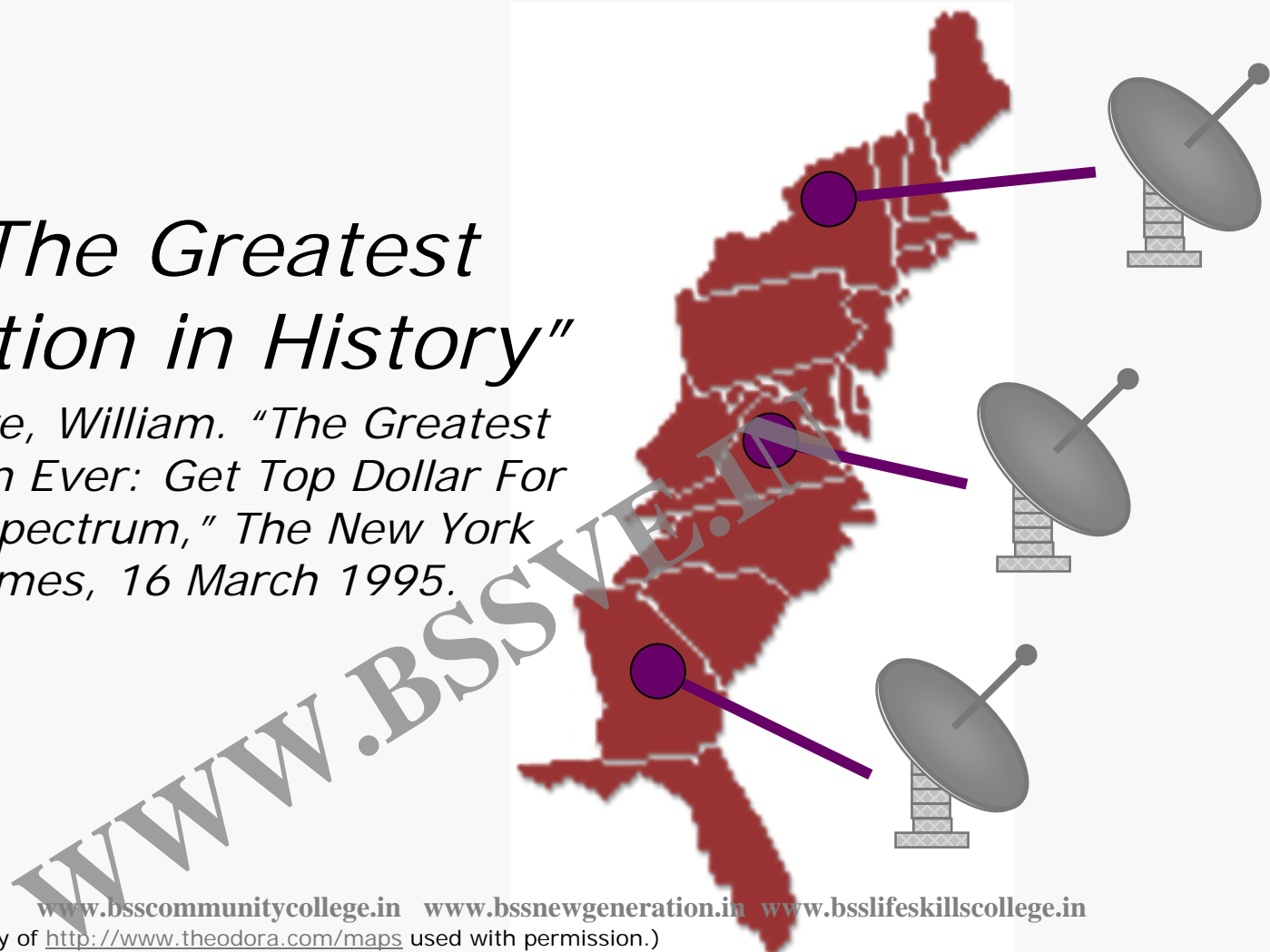
-Oscar Wilde, Lady Windemere's Fan, 1892

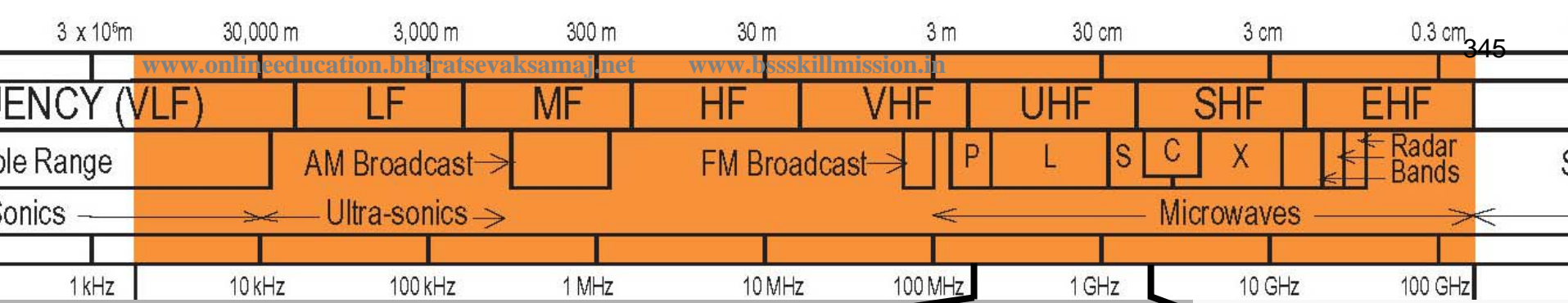
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PCS Spectrum Auctions

"The Greatest Auction in History"

- Safire, William. "The Greatest Auction Ever: Get Top Dollar For the Spectrum," *The New York Times*, 16 March 1995.





- 99 licenses (corresponding to the red circles) were sold to 18 companies for a total price of \$7.7 billion

Images of spectrum courtesy of U. S. Department of Commerce's National Telecommunications and Information Administration (<http://www.ntia.doc.gov/>, accessed 16 July 2004).

What is an Auction?

auc•tion

1. A public sale of property or merchandise to the highest bidder.
2. A market institution with explicit rules which determine prices and the allocation of resources based on bids.
3. Bidding in the game of bridge.

Derivation: From the Latin "auctus", which is the past participle of "augere", to increase.

Examples of Auctions

- FCC Spectrum
- Procurement
- Electricity generation
- Treasury Bills
- Internet
- Wine
- Quota Rights

Types of Bidders

- Auctions have *rules* and *bidders*
 - Auctioneer decides what rules to use but takes bidders (“the environment”) as given
- Two main types of bidders
 - private value
 - common value

Private Value

- Dinner

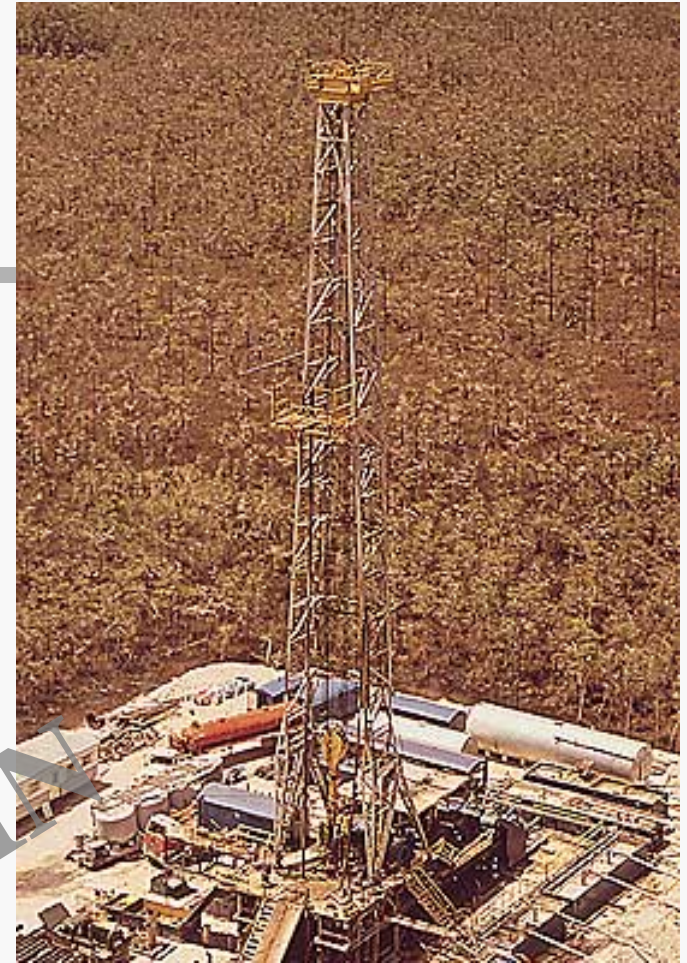


Source: Photograph courtesy of Erik Dungan,
<http://sxc.hu> (accessed August 10, 2004). Used with permission.

- What others know does not effect your willingness to pay

Common Value

- Unproven oil fields
- Object has *same* value to all bidders, but each only has an *estimate* of that common value



Source: Photograph No. 544512; "New Oil Rig, North of Gum Slough, in Big Cypress Swamp," August 1972; Still Picture Records LICON, Special Media Archives Services Division (NWCS-S), National Archives at College Park, MD.

Types of Rules: Open Outcry

Bidders interact ("call out bids"). Most common sorts:

1. English auction. Price increases until only one bidder remains
2. Dutch auction. Price decreases until some bidder jumps in
3. War of Attrition. War continues until only one bidder remains.

Types of Rules: Sealed Bid

Bidders tell auctioneer their bids without interacting with each other

1. First-price. Winner pays its own bid. Losers pay nothing.
2. Second-price. Winner pays highest losing bid. Losers pay nothing.
3. All-pay. Each bidder (including losers) pays its own bid.

“Auctions in Disguise”

Many interactions have the hallmarks of an auction:

1. There is a **prize**
2. Prize has **value** that is never less if others value it more
3. Each party makes a **bid** where highest bidder gets prize
4. Bidding has a **cost**, where higher bids don't cost less

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Hiring Decision

- McKinsey and Charles River are trying to recruit Sven
- Whoever makes the highest wage offer will get Sven
- What type of bidders?
- What type of rules?

Labor Dispute

- Labor and management have a dispute over new work rules
- Work stops until some side gives in
- What type of bidders?
- What type of rules?

Promotion Tournament

- Amande and Mert are contenders to become the firm's next CEO
- Whoever spends the most weekends in the office gets the job
- What type of bidders?
- What type of rules?

Competitive Negotiation

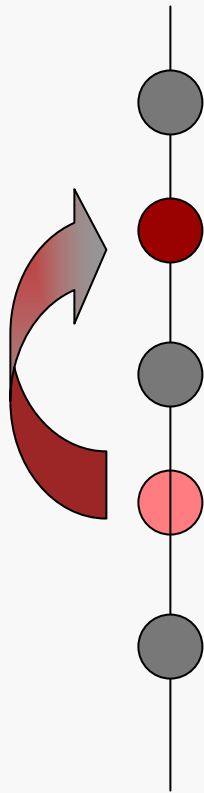
- Boeing and Airbus are each trying to get Iberia's business
- Iberia's CFO forces the two firms to continue beating each other's best offers and counteroffers until someone gives up
- What type of bidders?
- What type of rules?

Second Price Auction

- Bidding strategy is easy!
- Bidding one's true valuation is a (weakly) dominant strategy
- *Intuition:* your bid determines whether you win, not what you pay

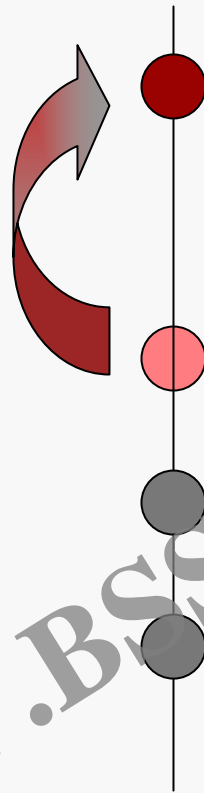
Bidding Higher Than My Valuation

Case 1



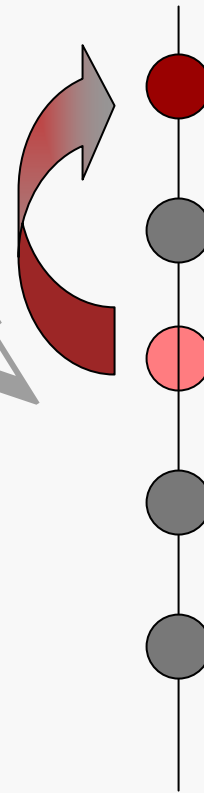
No difference

Case 2



No difference

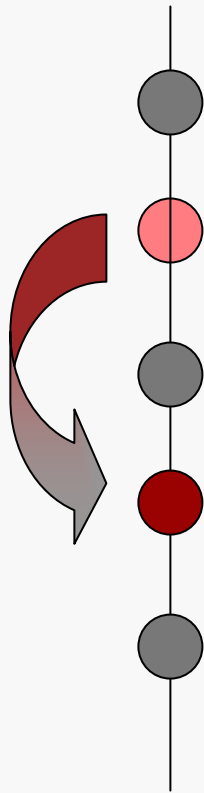
Case 3



Lose money

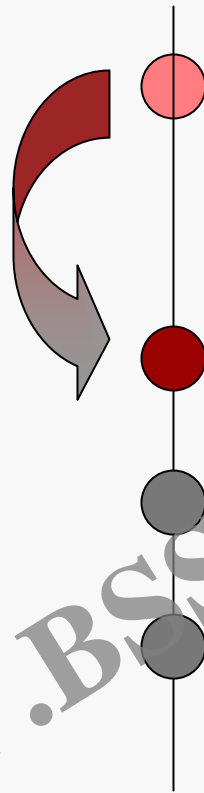
Bidding Lower Than My Valuation

Case 1



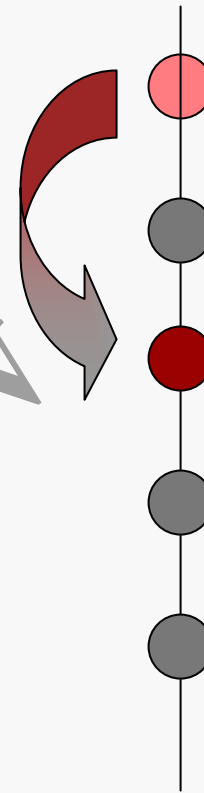
No difference

Case 2



No difference

Case 3



Lose money

First Price Auction

- Bidding your valuation guarantees you get no surplus
- But bidding lower risks regret, i.e. losing when willing to pay the winning bid
- *Optimal bid trades off risk of not winning vs. extra gain from winning with lower bid*

Revenue Equivalence

- Amazingly, there are many settings in which first-price, second-price and other auctions yield
 - same expected revenue for auctioneer
 - same expected surplus for each bidder

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Revenue Equivalence in Simple Example

- Suppose two bidders, each with private value random from 0 to 2
- First-price auction: turns out equilibrium is for each bidder to *bid half its value*.
 - $E[\text{revenue}] = 50\% \text{ of } E[\text{maximum}] = 2/3$
- Second-price auction: bid true value
 - $E[\text{Revenue}] = E[\text{minimum}] = 2/3$



Conditions for Revenue Equivalence

1. **Bidders play equilibrium strategies in both auctions**
2. **Bidders have private values that are not correlated with one another**
3. **Both auctions lead (in equilibrium) to the same allocation of the prize**
4. Bidders are risk neutral
5. A bidder with the lowest possible value gets zero surplus in both auctions

Usefulness of Revenue Equivalence Theorem (RET)

- If conditions of RET are met, then for given bidders it doesn't matter which auction you choose
 - *just focus on attracting more bidders!*
- But when some of the conditions fail (as they often do) RET gives insight into *why* one auction is likely better than another
 - better for auctioneer and/or better for bidders

What if private values fails?

- When private values fail, this means that bidders care about the information that others possess.
- Which is better, open auction or sealed-bid auction:
 - for bidders?
 - for the auctioneer?

What if there is correlation?

- When bidders' values are correlated, each expects others to have higher values when it has high value
- Which is better, first-price or second-price auction:
 - for bidders?
 - for the auctioneer?

What if they are risk averse?

- In this case, a stronger version of RET tells us that bidders must still get same expected utility from the two auctions
- Which is better, first-price or second-price auction:
 - for the auctioneer?

What if they don't follow equilibrium strategies?

- There are many ways that people might fail to play in equilibrium
- One is "animal spirits", i.e. you want to win more if someone else is trying to win
- Which is better, open or sealed-bid:
 - for bidders?
 - for the auctioneer?

What if they don't follow equilibrium strategies?

- Another way not to play in equilibrium is to collude, i.e. bidders try to maximize joint surplus rather than individual surplus
- Which is better, open or sealed-bid:
 - for bidders?
 - for the auctioneer?

Case Study: Iberia and Boeing vs. Airbus

- You are CFO of Iberia
- Boeing and Airbus both want your business
- What sort of auction is probably best for you, the auctioneer?

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On-Line Game #8

The Winner's Curse

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Uncertain Value

■ Uncertain Valuation

- A company is worth between \$0 and \$1000 per block of shares

■ Synergy

- Worth of company increases by 50% if purchased

■ Adverse Selection

- Offer only accepted if company is worth less than offer

Levels of Thinking

What would I be willing to pay given

what I know before submitting my bid

versus

what I know before submitting my bid,

and

that I will only win if no one else
is willing to bid higher than me

Avoiding Winner's Curse

- Since winning means you have the highest signal, always bid *as if* you have the highest signal
- If you have highest signal – what is the object worth?
- Use that as the basis of your bid

FCC Spectrum Auctions: Blocks C & F

- Gov't wanted to encourage minority and female-owned firms to bid but licenses are very expensive.
- Reserved "blocks C & F" for smaller bidders.
- Allowed 10% down, low interest, remaining principal owed in 7 years.
- What happened?

Adverse Selection and Blocks C & F

- Bid high – if licenses end up being worth less, default!
- 83 winners:

Of the four largest, ...

- ... went bankrupt and defaulted
- ... got \$1B reduced to \$66M in bankruptcy court
- ... was a front for Qualcomm
- ... was sold to Siemens

Summary

- *As auctioneer*, understand the bidders to determine which auction to use:
 - private vs. common values?
 - correlated signals?
 - risk-aversion?
 - collusion? animal spirits?
- *As bidder in common-value settings*, bear in mind the selection effect that you only win when others have bid less than you
- *Next time*: more on selection (signaling) as well as incentives

Online Game #7 (Incentives)

- Play Online Game #7 prior to midnight before next lecture.
- Note: We are *not* playing the games in their numerical order!!

Lecture 10 Game Plan

- Hidden actions, moral hazard, and incentives
- Hidden traits, adverse selection, and signaling/screening

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Hidden Information

"A little knowledge is a dangerous thing.

So is a lot."

- Albert Einstein

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Strategic Manipulation of Hidden Information

- Hidden Actions: Incentives
 - Associates others' unobservable actions with observable outcomes

- Hidden Traits: Signaling & Screening
 - Associates others' unobservable traits with their observable actions

Incentives

- High hurdle and a lot of money
- Low hurdle and a little money

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Hidden Effort

- You are contracting a project to an outside firm. The project has an uncertain outcome
 - Probability of success depends on firm's effort
 - prob. of success = 0.6 if effort is routine
 - prob. of success = 0.8 if effort is high
 - Firm has cost of effort
 - cost of routine effort = \$100,000
 - cost of high effort = \$150,000
 - Project outcome = \$600,000 if successful

Compensation Schemes

- I. Fixed Payment Scheme
- II. Observable Effort
- III. Bonus Scheme
- IV. Franchise Scheme

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Incentive Scheme 1: Fixed Payment Scheme

- If firm puts in routine effort:
 - Profit = Payment - \$100,000
- If firm puts in high effort:
 - Profit = Payment - \$150,000
- Firm puts in low effort!
→ *“moral hazard”*
- Optimal Payment: lowest possible.
 - Payment = \$100,000
- Expected Profit
= (.6)600,000 - \$100 = \$260K

Incentive Scheme 2

Observable Effort

- Firm puts in the effort level promised, given its pay
- Pay \$100,000 for routine effort:
 - $E[\text{Profit}] = (.6)600,000 - 100,000$
 $= \$260,000$
- Pay additional \$50K for high effort:
 - $E[\text{Profit}] = (.8)600,000 - 150,000$
 $= \$330,000$
 - want to induce high effort
- Expected Profit = \$330K

Problems

- Fixed payment scheme offers no incentives for high effort
 - High effort is more profitable

- Effort-based scheme cannot be implemented
 - Cannot monitor firm effort

Incentive Scheme 3

Wage and Bonus

- Suppose effort can not be observed
- Compensation contract must rely on something that can be directly observed and verified.
 - Project's success or failure
 - Related *probabilistically* to effort
 - Imperfect information

Salary + Bonus Schemes

A successful scheme must

1. Be “Incentive Compatible”

- Firm must *prefer* to put in high effort

2. Induce Participation

- Firm must *prefer* to take the job

On-Line Game #7

Incentive Pay

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Incentives

- Cost of routine effort: \$100K
- Cost of high effort: \$150K
- Added cost of high effort: \$50K

- Benefit of routine effort: .6b
- Benefit of high effort: .8b
- Added benefit of high effort: .2b

Incentive Compatibility

- Firm will put in high effort if

$$s + (0.8)b - 150,000 \\ \geq s + (0.6)b - 100,000$$

- $(0.2)b \geq 50,000$

marginal benefit > marginal cost

- $b \geq \$250,000$

Participation

- Expected salary must be large enough to make work worthwhile
- If induce high effort: $b > \$250K$
expected salary = $s + .8b$
but even if $s = 0$:
 $.8b = \$200K > \$150K$
- No base salary needed!

Profitability Summary

- Greatest Profit from inducing high effort:
\$280K (unless $s < 0$)
- Greatest Profit from inducing low effort:
\$260K
 - Using the “no brainer” solution
 - Salary = \$100K, no bonus
- Do we want to induce high effort?
- Carefully.
- Don't give away the farm to do it.

Optimal Salary and Bonus

- Incentive Compatibility:
 - Firm will put in high effort if $b \geq \$250,000$

- Participation:
 - Firm will accept contract if $s + (0.8)b \geq 150,000$

- Solution
 - Minimum bonus: $b = \$250,000$
 - Minimum base salary:
 $s = 150,000 - (0.8)250,000 = -\$50,000$

Negative Salaries?

- Ante in gambling
- Law firms / partnerships
- Work bonds / construction
- Startup funds

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Interpretation

- \$50,000 is the amount of capital the firm must put up for the project
- \$50,000 is the fine the firm must pay if the project fails.

- Expected profit:

$$\begin{aligned} & (.8)600,000 - (.8)b - s \\ &= (.8)600,000 - (.8)250,000 + 50,000 \\ &= \$330,000 \end{aligned}$$

- Same as with observable effort!!!

Incentive Scheme 4

Franchising

- Charge the firm f regardless of profits
 - Contractee takes all the risks and becomes the “residual owner” or franchisee
- Charge franchise fee equal to highest expected profit
 - Routine effort: $.6(600K) - 100K = 260K$
 - High effort: $.8(600K) - 150K = 330K$
- Expected Profit: \$330K

Summary of Incentive Schemes

■ Observable Effort

- Expected Profit: 330K
- Expected Salary: 150K

■ Salary and Bonus

- Expected Profit: 330K
- Expected Salary: 150K

■ Franchising

- Expected Profit: 330K
- Expected Salary: 150K

Upside of Assigning Risk

- Assign risk to the *agent*, the party that has control of the hidden action
- This leads to
 - more efficient outcome
 - more profit for the principal

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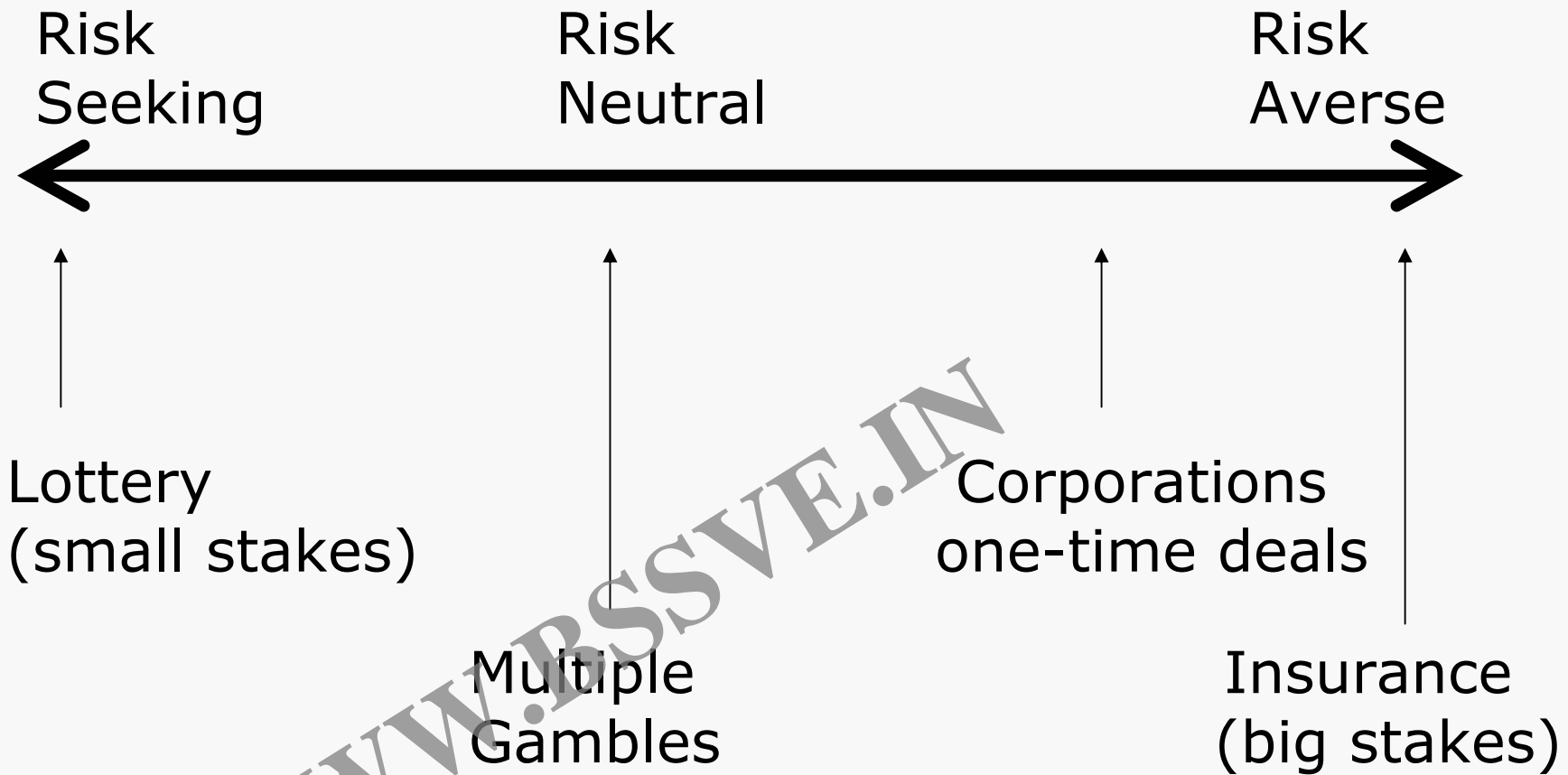
Downside of Assigning Risk

- Employees (unlike firms) are rarely willing to bare high risks

- Salary and Bonus
 - 0.8 chance: 200K
 - 0.2 chance: -50K

- Franchising
 - 0.8 chance: 270K
 - 0.2 chance: -330K

Risk Aversion



Summary So Far

- Suppose you know agent's payoffs but *can't observe its actions*.
- You can still induce agent to take action you want by making it bear more risk
 - Franchising
 - Salary and bonus
- Such schemes can give as much profit as if you could observe actions perfectly!

Venture Capital

- A venture's success depends on whether a new technology will work
 - 50% chance it works
 - venture worth \$20M if it works
 - venture worth \$0 if it doesn't work
- Entrepreneur knows whether the technology works or not

Venture Capital

- Entrepreneur approaches you: “I am somewhat risk averse and hence prefer to take a smaller than 100% stake”
- How much are you willing to pay if she offers you
 - 50% stake?
 - 90% stake?

Problem of Adverse Selection

- Expected value of venture given that she wants to sell 50%
 - $(50\% * 20 + 50\% * 0) = \$10M$
- Expected value of venture given that she wants to sell 90%
 - $100\% * 0 = \$0M$
- Because of this “adverse selection”, you are willing to pay *less* for a larger stake!!

Problem of Average Selection

- Only “bad” entrepreneur is willing to sell 90% of venture
 - adverse selection if you buy 90%
- But both “good” and “bad” are willing to sell 50% of venture
 - average selection if you buy 50%
- Still not ideal: you only want to invest when technology works!

Signaling & Screening

Screen = "Jump over this while I watch"

Signal = "Watch while I jump over this"

- High hurdle and a lot of money
- Low hurdle and a little money

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How to Screen

- Want to know an *unobservable* trait
- Identify a “hurdle” such that:
 - those who jump the hurdle get some benefit but at some cost
 - “good” types find the benefit exceeds the cost
 - “bad” types find the cost exceeds the benefit
- This way we get **self-selection**: only “good” types will jump the hurdle

Auto Insurance

- Hidden Trait = high or low risk?
 - Half of the population are high risk, half are low risk
 - High risk drivers:
 - 90% chance of accident
 - Low risk drivers:
 - 10% chance of accident
 - Accidents cost \$10,000

Example: Auto Insurance

- The insurance company can not tell who is high or low risk
- Expected cost of accidents:
 - $(\frac{1}{2} \cdot .9 + \frac{1}{2} \cdot .1)10,000 = \$5,000$
- Offer \$6,000 premium contract to make \$1,000 profit per customer
- What happens?

Self-Selection

- High risk drivers:
 - Don't buy insurance: $(.9)(-10,000) = -9K$
 - Buy insurance: $= -6K$
 - High risk drivers buy insurance
- Low-risk drivers:
 - Don't buy insurance: $(.1)(-10,000) = -1K$
 - Buy insurance: $= -6K$
 - Low risk drivers do not buy insurance
- Only high risk drivers buy insurance

Adverse Selection

- Expected cost of accidents in population
 - $(\frac{1}{2} \cdot .9 + \frac{1}{2} \cdot .1) 10,000 = \$5,000$
- Expected cost of accidents among insured
 - $.9 (10,000) = \$9,000$
 - Insurance company loss: \$3,000
- Cannot ignore this “adverse selection”
- If only going to have high risk drivers, might as well charge more (\$9,000)

Screening

- Offer two contracts, so that the customers self-select
- Compare contracts aimed at high- and low-risk drivers.
 - Which will have the higher premium?
 - Which will have the higher deductible?

“New Issues Puzzle”

- Firms conducting seasoned equity offerings (SEOs) afterwards perform worse on average than other firms
- Loughran and Ritter (J Finance 1995) argue you lose 30% over five years investing in a SEO
- 1970-1990 data. Comparison is relative to performance of “matched firm”, i.e. one having similar characteristics that did not have any SEO in the following 5 years

SEO Underperformance

For this table, please see Table II from:

Loughran, Tim, and Jay Ritter. "The New Issues Puzzle" *Journal of Finance* 50, no. 1 (1995): 23-51.

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Is the market failing?

- Why doesn't the market assimilate this information immediately?
- One possible explanation: *positive selection*
 - "Matched firms" are chosen retrospectively to be firms that will not have any SEO in *next* five years
 - Even if the market had *already* priced in the negative info, it might not have assimilated the (future) positive info about the matched firm!

Signaling

- The seasoned offering is a signal about the status of the companies current projects as well as future ones.

Seek outside equity



Fund projects internally



LOW

HIGH

Profitability of current/future projects

... & Adverse Selection

- If the current projects are not profitable, the cost (in dilution) to the owner-manager of issuing new share is lower.
- Therefore, seasoned offering is likely associated with
 - bad news about the firm's present condition
 - low threshold for profitability of new project.

Dividends

“It would be uneconomic as well as pointless [for firms to pay dividends and raise capital simultaneously]”

- Merton Miller and Kevin Rock, 1982

Dividends

- Why might it be make sense for a firm to issue a dividend *and* for investors to view this positively?

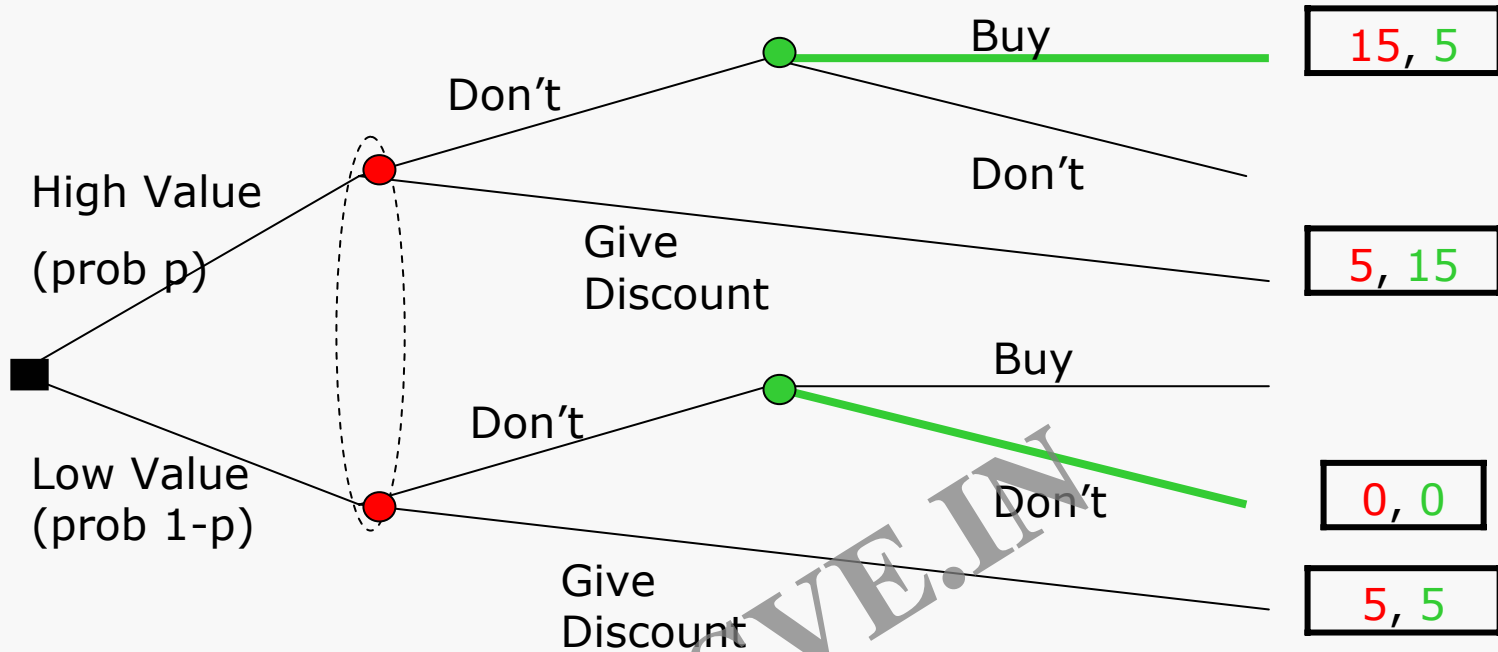
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Bargaining with a Customer

- Customer either willing to pay \$20 or \$10, *equally likely*
- Your price is \$15 (zero costs), but customer asks for a deeply discounted price of \$5
- You don't know whether the customer has value \$20 or \$10

Solving for "Sequential Eqm"



Seller's equilibrium choice depends on its **belief** about likelihood of High Value vs. Low Value

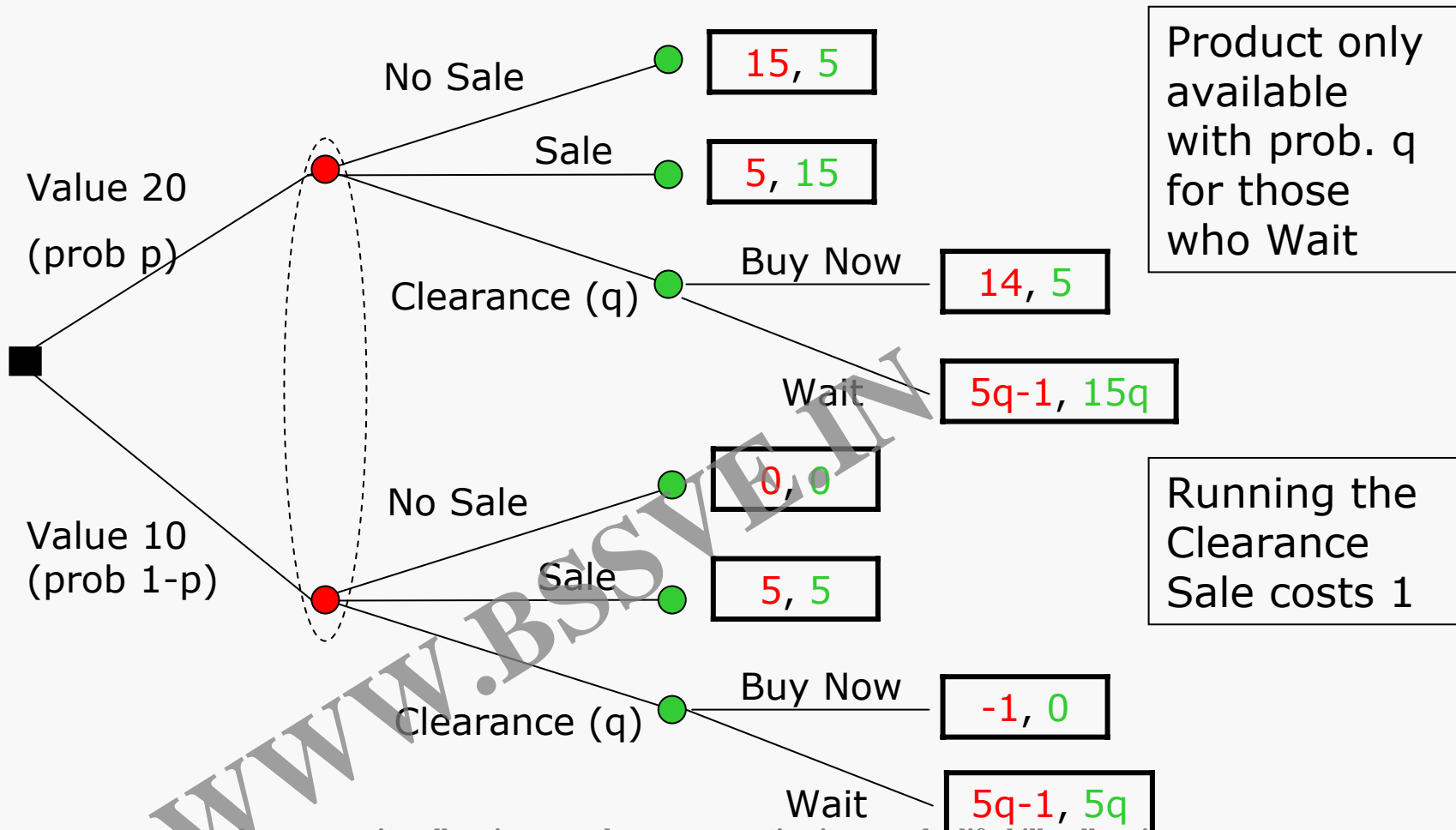
- *By Don't Discount, seller is "risking 5 to gain 10"*
- **Don't Discount if $p > 1/3$**

Other Approaches?

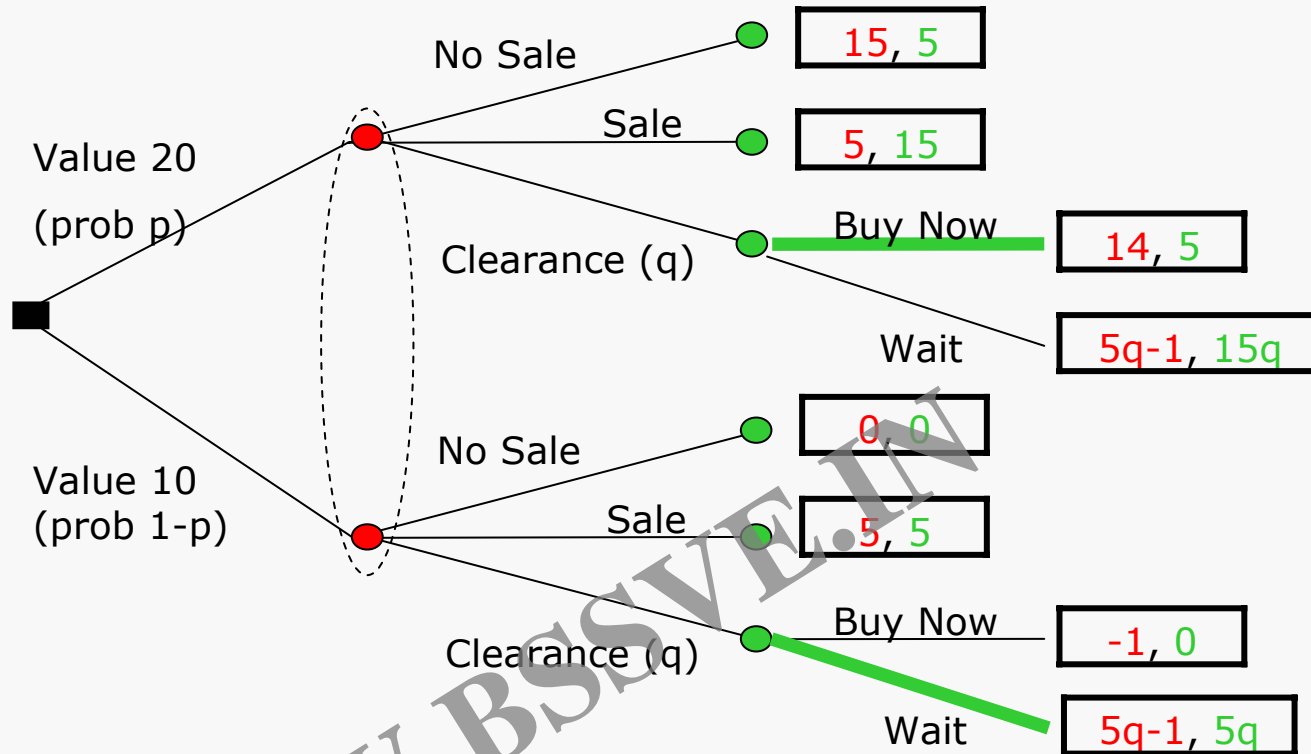
- If a customer “pleads poverty” for a discount, you have other options than simply to grant/refuse request
- What else might you do?

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Clearance Sale



Clearance Sale as Screen



Clearance is an *effective screen* if $q < 1/3$

Clearance Sale?

Clearance Sale
or Sale?

Clearance Sale
or No Sale?



- $p > 1/3$: No Sale better than Sale
- $p < 1/3$: Sale better than No Sale

When (not) to have Clearance Sale ($p < 1/3$)

Clearance Sale
or Sale?

Clearance Sale
or No Sale?



- Clearance Sale vs. Sale
 - Clearance gives +9 more on High
 - Clearance loses $1 + 5(1-q)$ on Low
- *Only have Clearance when chance of High is sufficiently large*

When (not) to have Clearance Sale ($p > 1/3$)

Clearance Sale
or Sale?

Clearance Sale
or No Sale?



- Clearance Sale vs. No Sale
 - Clearance gives $-1 + 5q$ more on Low
 - Clearance loses 1 on High
- *Only have Clearance when chance of High is sufficiently low*

When to have Clearance Sale ($p = 1/3$)

Clearance Sale
or Sale?

Clearance Sale
or No Sale?



- If Clearance is *ever* your best strategy, it must be *when you are indifferent* between Sale and No Sale ($p = 1/3$)
 - *“when you can’t decide whether to offer a High- or Low-Quality product, offer both!!”*

Versioning

- Suppose that high-quality/high-cost item will be equally profitable as low-quality/low-cost item
- In this case, you can always do better offering a *menu* of both items that acts as a consumer screen

Versioning: Example

Customer willingness -to-pay	GOOD PRODUCT	BAD PRODUCT
HIGH CUSTOMER	\$35	\$20
LOW CUSTOMER	\$20	\$15

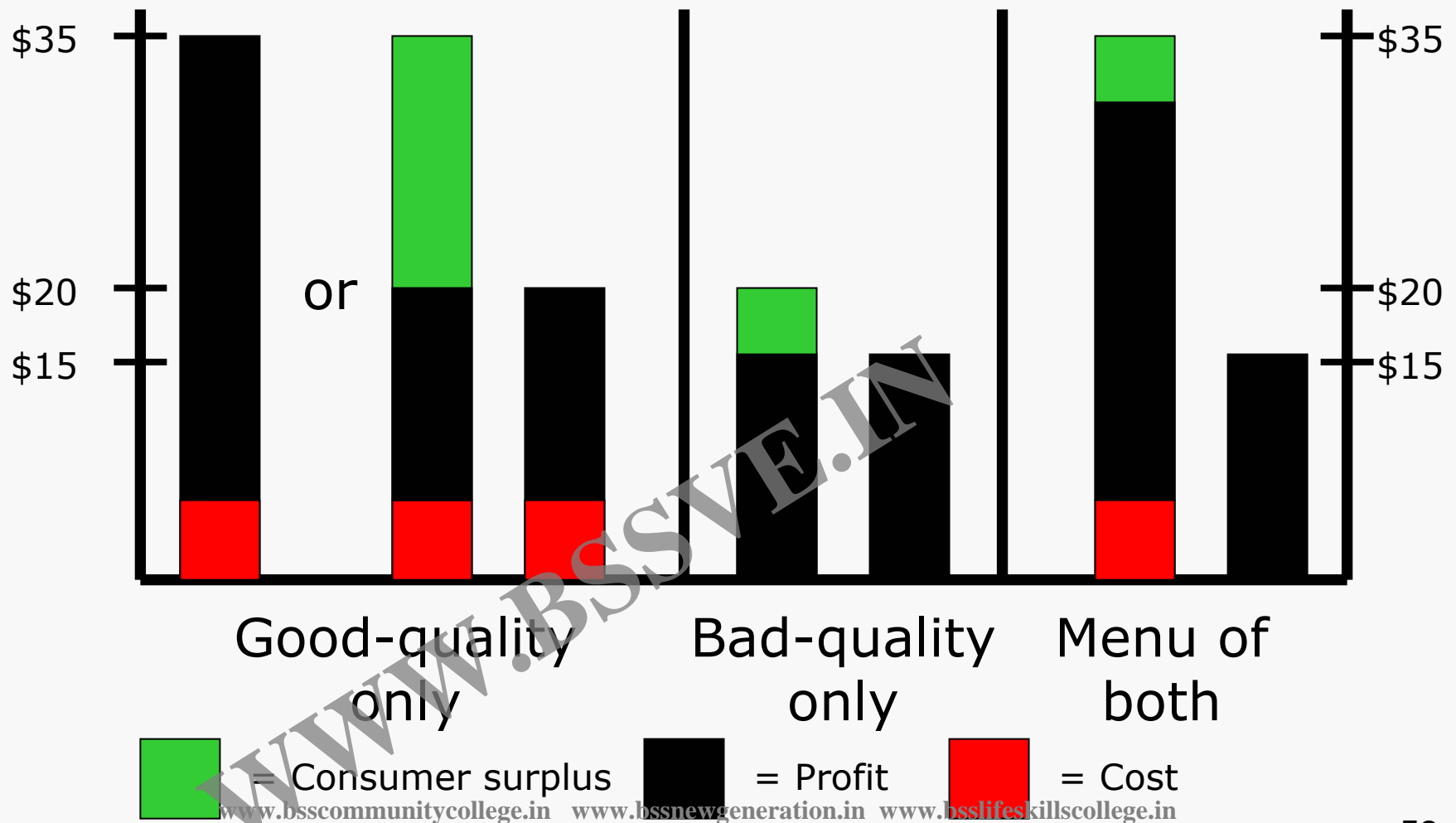
Good product costs \$5, bad product \$0

Versioning: Example

	GOOD PRODUCT	BAD PRODUCT
HIGH CUSTOMER	\$35	\$20
LOW CUSTOMER	\$20	\$15

- Sell only Good $\rightarrow 2 * (\$20 - \$5)$ or $(\$35 - \$5)$
- Sell only Bad $\rightarrow 2 * (\$15 - \$0)$
- Sell both $\rightarrow (\$15 - \$0) + (\$30 - \$5)$

Good-quality vs. Bad-quality



Summary

- Strategic issues arise when different players have different information
- Moral hazard given hidden action
 - role for incentives / tying one's hands
- Adverse selection given hidden trait
 - role for screening / signaling
- *Next time*: using hidden traits about yourself to make a credible commitment

Lecture 11 Game Plan

- Reputation and “strategic irrationality”
- Course feedback

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Credibility and Reputation

*"This is our very soul ... our whole life.
For them, it's just another field."*

- Edwin Land, Polaroid founder, reacting to Kodak's entry into instant photography

Source: Dixit, Avinash, and Barry Nalebuff. *Thinking Strategically: A Competitive Edge in Business, Politics, and Everyday Life*. W. W. Norton & Company, 1991.

Kodak's assessment

- Kodak took care to enter gently
 - Kodak film and cameras were incompatible with Polaroid's
 - So Polaroid could still milk profits from its established customer base
- With Kodak's deep pockets, there was no way they would leave
- Thus, Polaroid was going to give in

Polaroid's Growl

- Polaroid responded by not giving in immediately, and saying that it *would never* give
- How should Kodak react to such statements? Why?

Growling Resolve

- Maybe growling changes Polaroid's payoffs enough that it *wants* to fight

See Figure 23.3 in:

Kreps, David M. *Microeconomics for Managers*. W.W. Norton & Company, 2004: 556-73. ISBN: 0393976793.

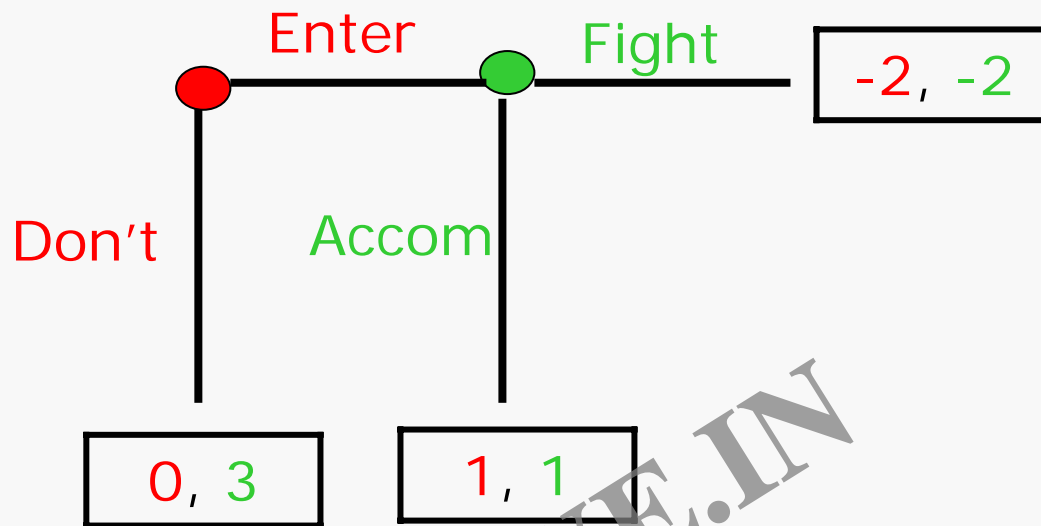
Growling Semi-Resolve

- What if growling doesn't change Polaroid's payoffs enough that it wants to fight?

See Figure 23.4 in:

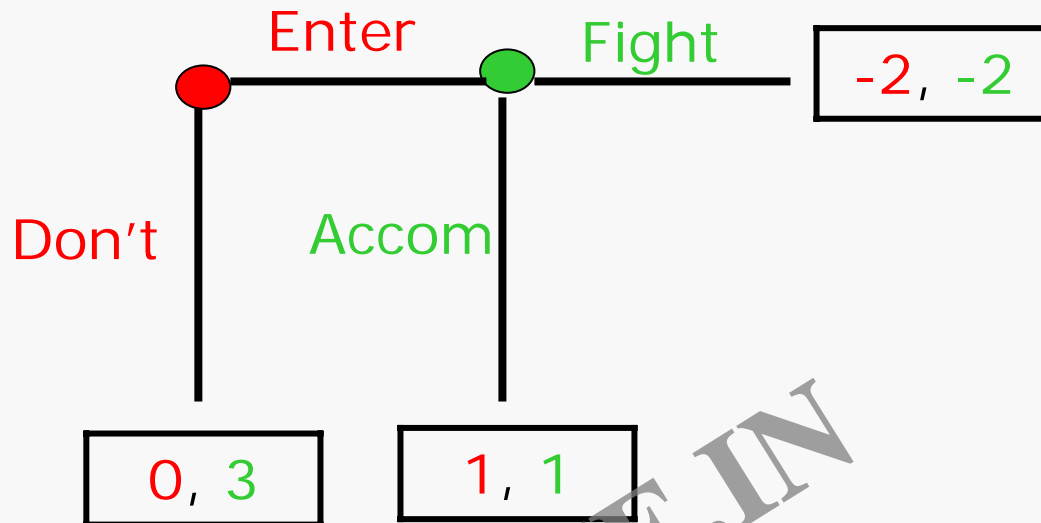
Kreps, David M. *Microeconomics for Managers*. W.W. Norton & Company, 2004: 556-73. ISBN: 0393976793.

Reputation in Repeated Games



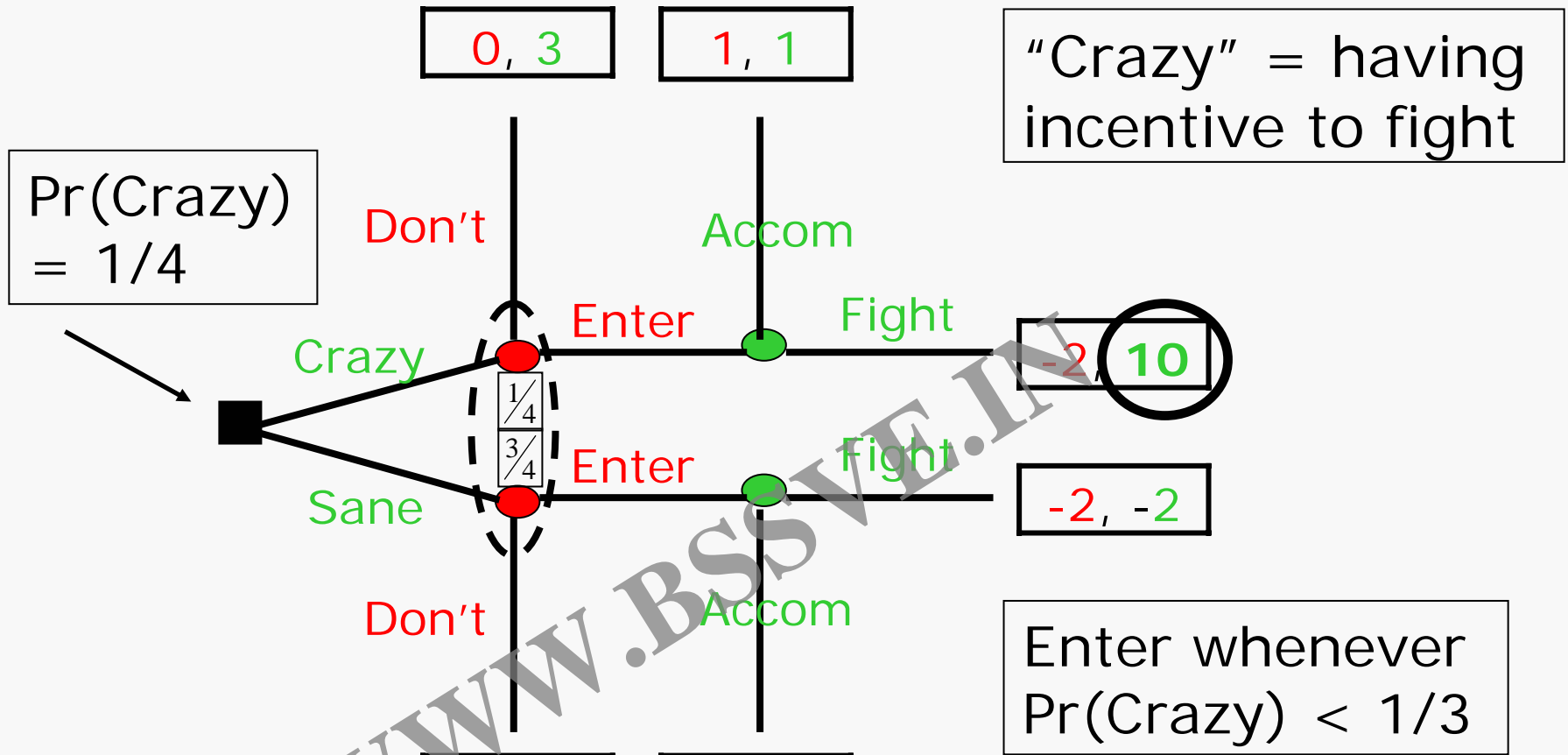
- Suppose incumbent monopolist always faces 80% chance that another entrant will come along
- Does this change things?

Reputation in Repeated Games

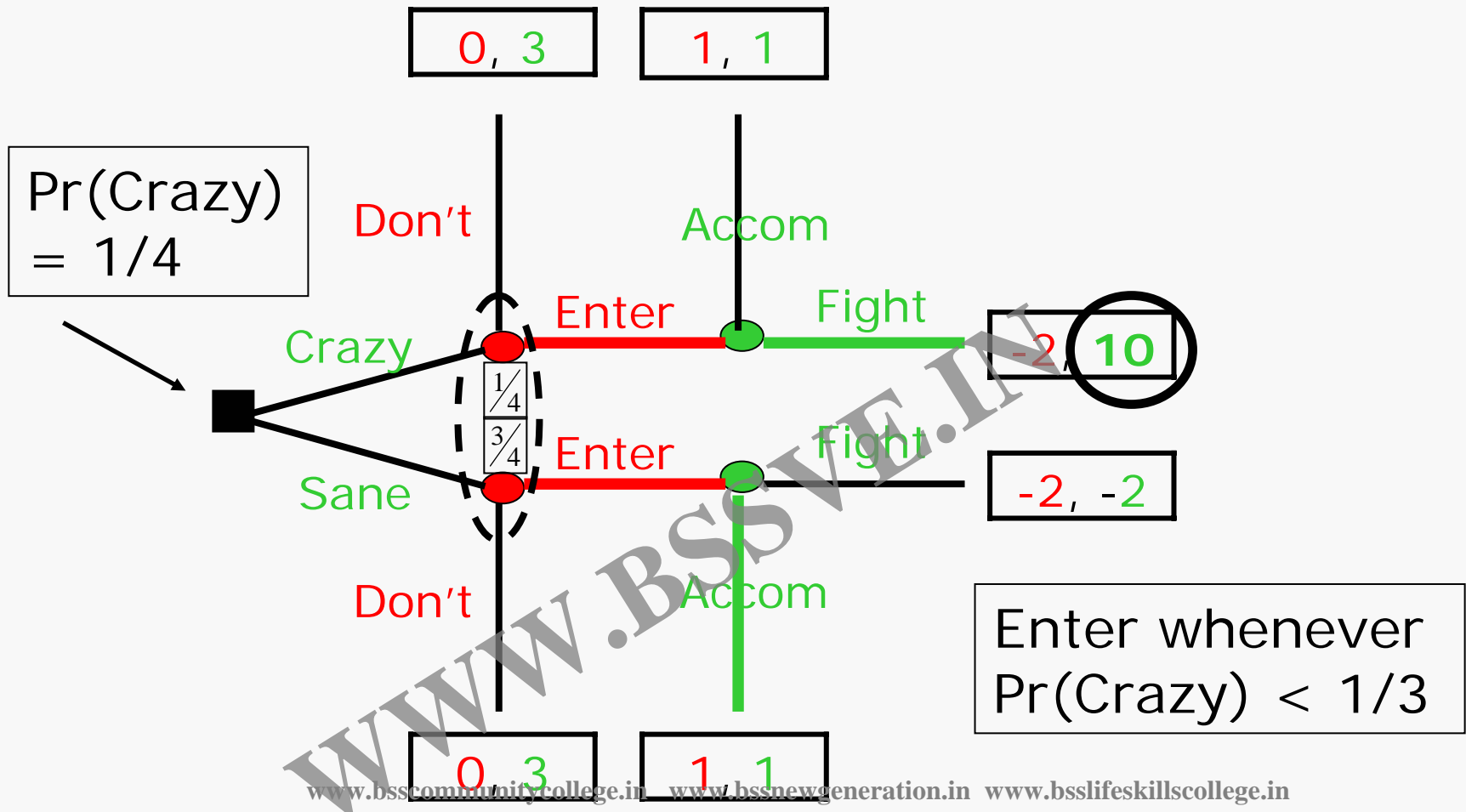


- Fighting **costs 3 today**
- If Fighting deters next entrant and the one after that, etc.. it has **benefit 2 in every future period**
 - benefit $2(.8 + .8^2 + \dots) = 2(1/.2 - 1) = +8$
- Fighting rational if it preserves "reputation"

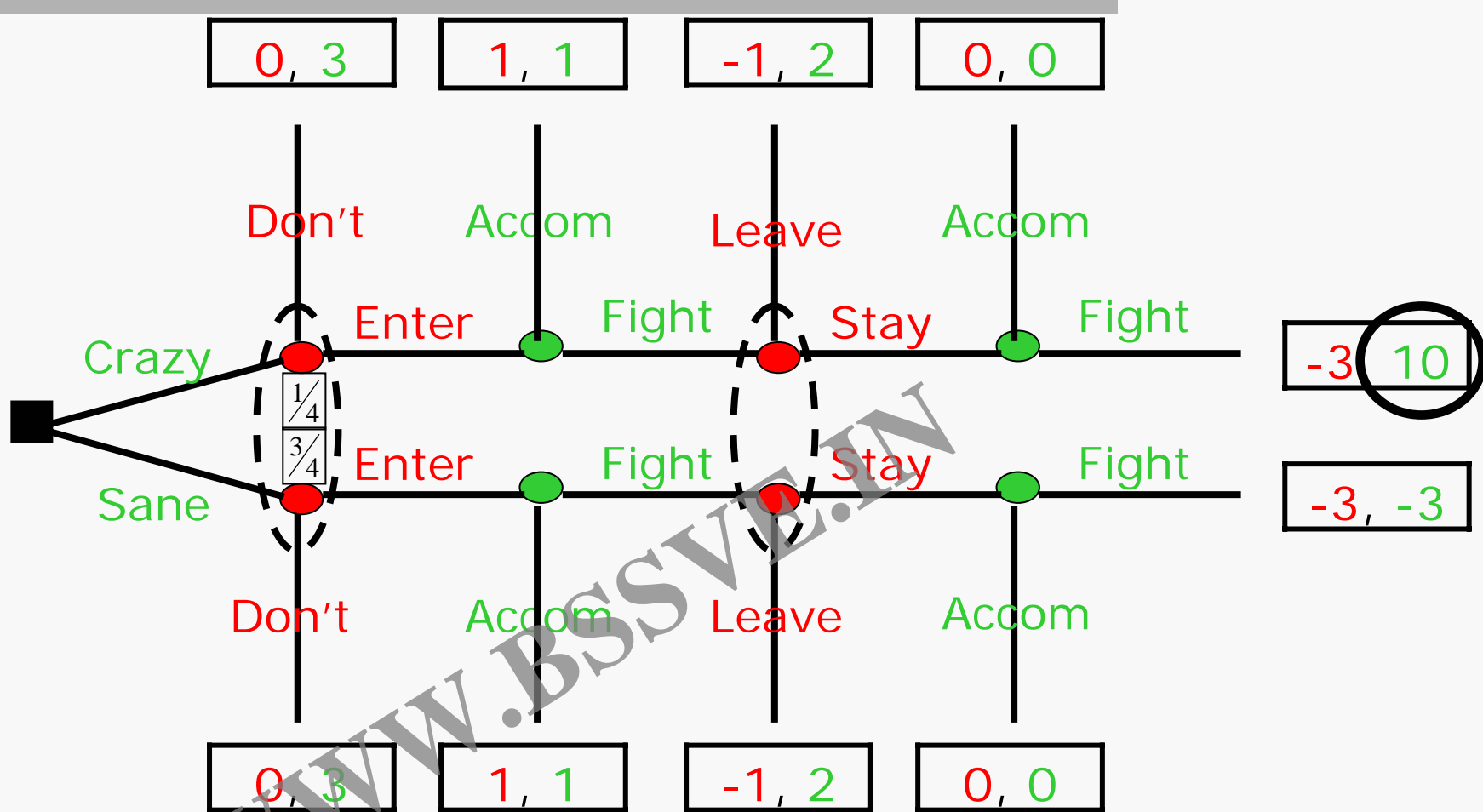
"Craziness" to Deter Entry



Not Crazy Enough ...



Responding to "Craziness"



- What is the sequential equilibrium of this game?

In-Class Game

Crazy(?) Incumbent Game

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Crazy(?) Incumbent: Rules

- Incumbent learns whether it is “Crazy” or “Sane”
 - “Crazy” incumbents love to fight
- Sequence of decisions
 1. Entrant – Enter?
 2. If so, Incumbent – Fight?
 3. If so, Entrant – Stay?
 4. If so, Incumbent – Fight?
- In last stage, only Crazy want to Fight.

Crazy(?) Incumbent: Payoffs

- See handout for details

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Get Crazy!

We will play in sets of five people

- one Entrant
- four Incumbents

The Entrant may Enter any/all of the four markets

One of the four Incumbents is Crazy!

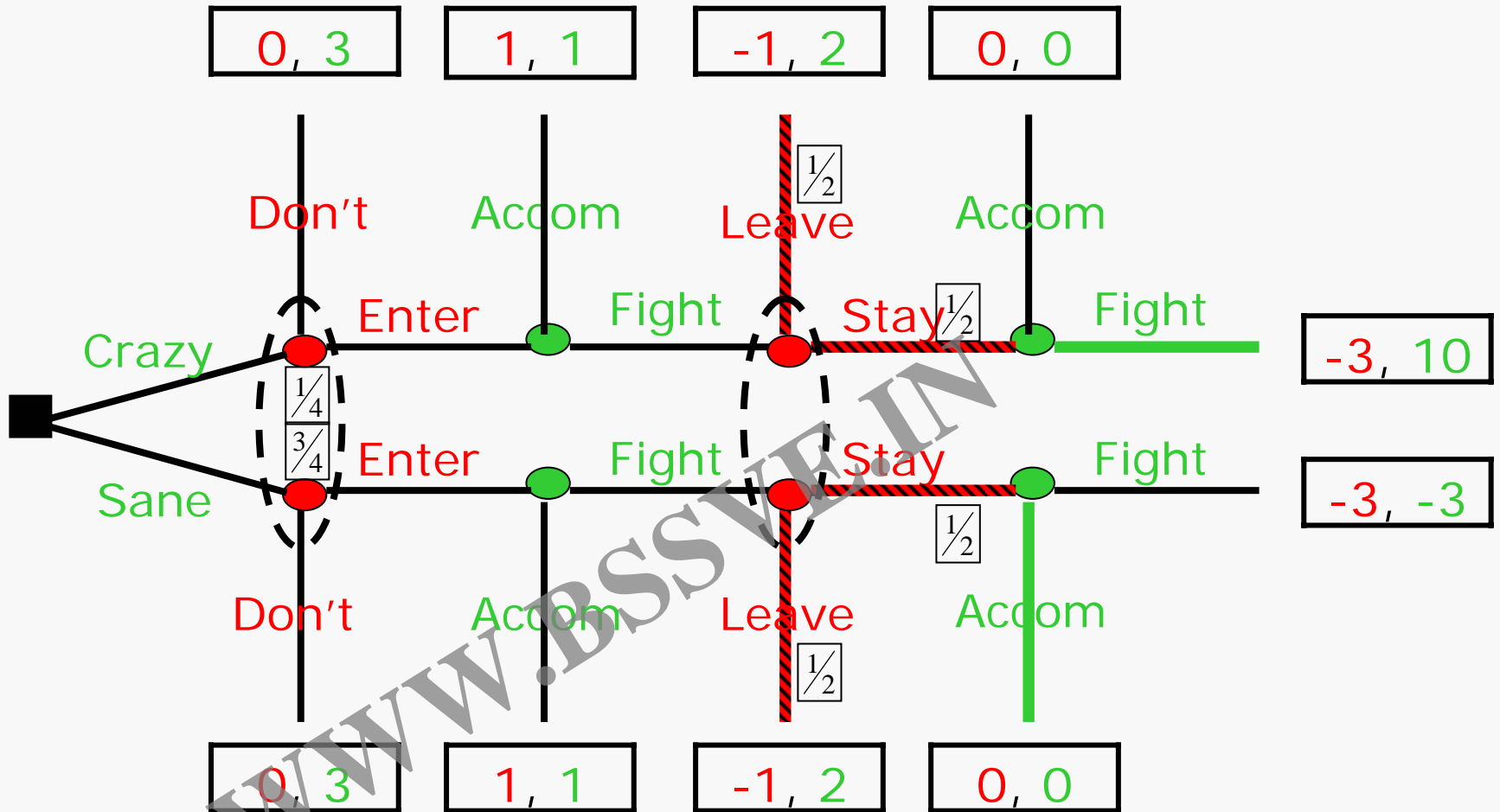
Get Crazy!

- First, each Incumbent will learn whether it is Crazy or Sane
 - not allowed to credibly reveal this
- Next, Entrant decides which (if any) of the four markets to enter
- In markets with entry, Incumbents decide whether to fight
- In markets with fighting, Entrant decides whether to stay
- In markets with Entrant staying, Incumbent decides whether to Fight

Sane Act Crazy

- Some (but not all!) Sane must Fight
 - if all Fight, entrant will Stay for sure
 - if none Fight, entrant will Leave for sure
- So, Sane must be indifferent to Fighting
- This requires that the entrant must Leave with probability $1/2$ after Fight

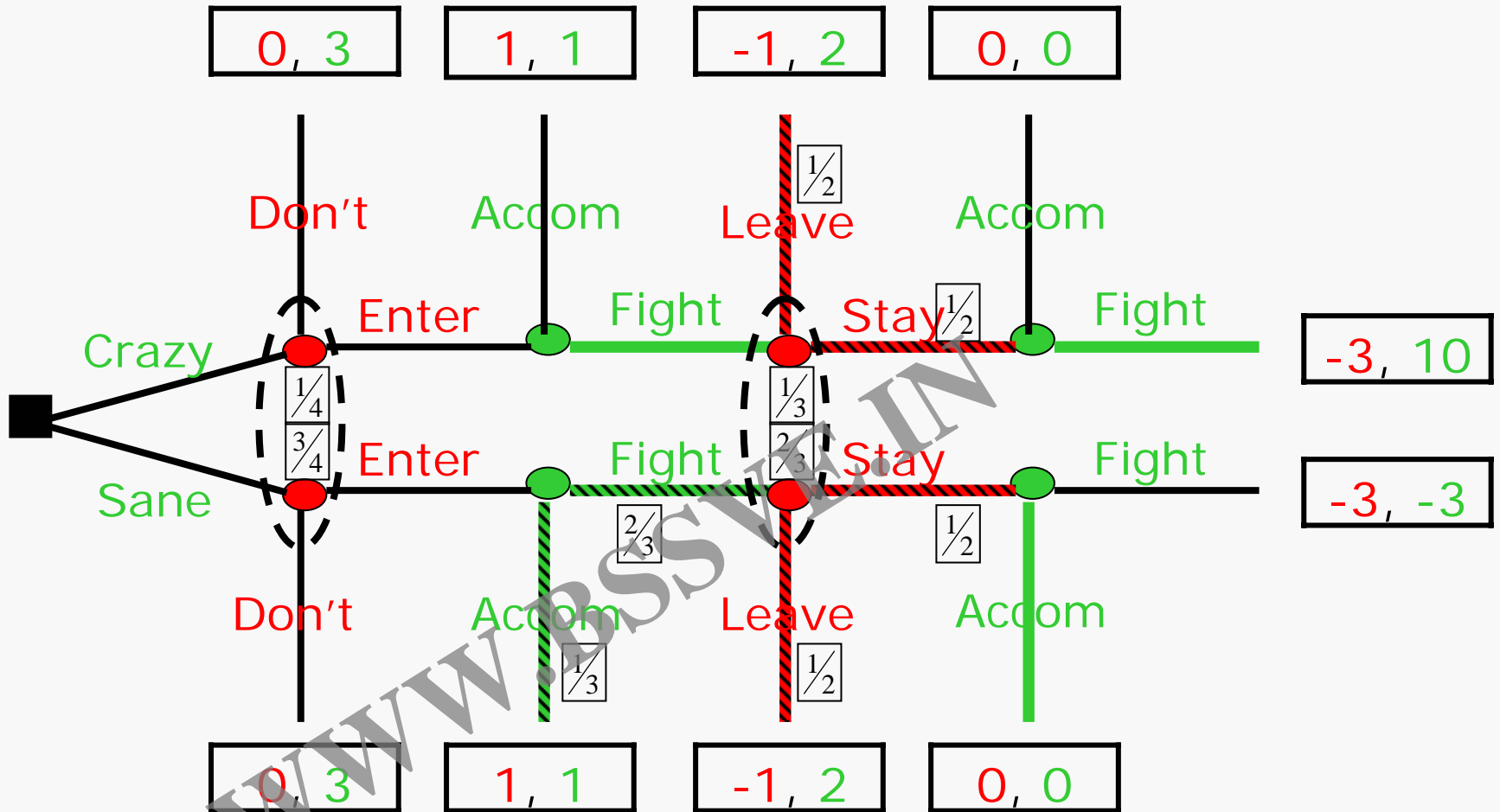
The Million Dollar Question: Will Entrant Enter?



Entrants Blink

- Some (but not all!) entrants must Leave
 - if all Leave, Sane will Fight for sure
 - if none Leave, Sane will Accom for sure
- Entrant must be indifferent to Leaving
- This requires that incumbent be Crazy with probability $1/3$ *conditional on Fight*
 - so 50% of 75% Sane will Fight

The Million Dollar Question: Will Entrant Enter?



Entrant Stays Out

- What happens if entrant enters?
 - all Crazy Fight (25%) plus 2/3 of the Sane Fight (50%)
 - *only get Accomodation 25%*
- Entrant should stay out
- What tips the balance is that Entrant must worry about both:
 1. Crazy opponents
 2. Sane opponents *acting* crazy (!)

Summary

- Reputation can help establish credibility in repeated games if losing reputation is costly:
 - lost "mystique"
 - others know you aren't Crazy

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**Office of the Vice President
Strategic Planning and Analysis
American Airlines, AMRCorp.**

American Airlines

April 10, 2003

Strategic Analysis Group
Eastern United States Division
Attention: Chief Strategist

Re: Revenue impact of extending “More Room” to Super 80s?

In recent years, as you know, we have come under sustained and increasing pricing pressure. Southwest and other discount airlines have expanded into many of our bread-and-butter routes and web price search engines have further sensitized many passengers to price. But we can't compete with the Southwests of the world purely on price. On a route like Baltimore-St. Louis, for example, our marginal costs are about \$50 per passenger while Southwest's are about \$20. Consequently, in late 2000 we introduced “More Room Throughout Coach”, an ongoing marketing campaign that highlights a retrofit of some aircraft in our fleet (mostly those flying longer routes) to have fewer seats in coach. Now, we are exploring whether to expand “More Room” to models such as the Boeing MD-80 serving shorter routes.

We've selected Baltimore-St. Louis as one of a few representative routes to study in more depth. American and Southwest both fly this route, with similar schedules – Southwest has seven 737-300 flights per day (137 seats each), we have five MD-80s (139 seats each). The folks upstairs hired a team from KPMG to come in and make recommendations, but I haven't been impressed with their work so far and want your team to get on this as well. So far, they've collected time series load data for this route at various price points to estimate the expected load per plane for both Southwest and American for given Coach prices ranging from \$100 to \$500 per leg on average round-trip Coach fares. (See attached tables.) Currently, Southwest charges \$150 while we charge \$200.

There are several issues that I need you to address. First, are these prices sustainable in the future at current capacities? Should we expect strengthening, stable, or eroding prices given current capacities? Also, I'm worried about the worst case scenario in which passengers are not willing to pay extra for the additional leg-room. In this worst case, how much will it cost us in revenue per flight to reduce the number of seats that we offer by 10% (to 125 seats)? It will be important, of course, to think about how Southwest is likely to respond in terms of pricing. To keep things from getting too complicated, please just assume that Southwest does not retrofit its cabins.

Thank you, sincerely,

Roger A. Pembroke
Vice-President, SP&A

Table I
Average number of passengers on Southwest Airlines flights

		Price Charged by American Airlines								
		100	150	200	250	300	350	400	450	500
Price Charged by Southwest Airlines	100	137	137	137	137	137	137	137	137	137
	150	73	90	120	132	134	137	137	137	137
	200	24	35	78	117	131	134	136	137	137
	250	2	5	18	77	113	119	129	131	132
	300	0	1	4	11	75	106	113	122	126
	350	0	0	1	3	9	47	84	99	118
	400	0	0	0	0	1	2	19	65	98
	450	0	0	0	0	0	0	0	9	55
	500	0	0	0	0	0	0	0	0	7

Table II
Average number of passengers on American Airlines flights

		Price Charged by Southwest Airlines								
		100	150	200	250	300	350	400	450	500
Price Charged by American Airlines	100	139	139	139	139	139	139	139	139	139
	150	125	139	139	139	139	139	139	139	139
	200	108	134	132	139	139	139	139	139	139
	250	67	99	115	129	139	139	139	139	139
	300	45	50	96	114	127	138	139	139	139
	350	14	15	16	17	25	116	137	139	139
	400	13	13	14	15	16	19	120	135	139
	450	12	12	12	13	14	14	17	89	135
	500	10	10	10	10	11	12	13	15	39

Data tables courtesy of Mike Shor, Vanderbilt University.

Sloan School of Management
Massachusetts Institute of Technology
Spring 2004, H1



Real-World Application

DUE DATE: This team assignment is due by the beginning of class on March 9th.

INSTRUCTIONS

Choose one of the four options below. **Provide: Two- to three-page memo** (plus any supporting tables, graphs, calculations, etc..) addressed to the relevant party in the game.

1. *Mixed Strategies (discussed in Lectures 3-4)*

Apply one of the games of Chapter 5 to a real-world application. Choose a game without an equilibrium in pure strategies and derive the mixed strategy equilibrium. From this analysis, develop strategy advice for one of the players in the game or for a party interested in the outcome.

2. *Sequential / Repeated Games (discussed in Lectures 3-6)*

Apply one of the games of Chapter 3 or Chapter 8 to a real-world application. The game should have a temporal element. That is, it should either be a sequential game or a repeated game. Make sure to identify the length of the game as well as the players, strategies, and payoffs. From this analysis, develop strategy advice for one of the players in the game or for a party interested in the outcome.

3. *Strategic Moves (discussed in Lectures 5-8)*

Apply one of the games in Chapter 9 to a real-world application. Discuss how the parties might benefit from making / anticipating a strategic move. If relevant, address issues of observability, irreversibility, commitment, credibility, and how these may be achieved. From this analysis, develop strategy advice for one of the players in the game or for a party interested in the outcome.

4. *Information (discussed in Lectures 8-10)¹*

Describe an asymmetric information problem. Identify the source of the asymmetry and the information possessed by each party. How could signaling or screening help resolve this uncertainty? Is this a profitable strategy? Why or why not? From this analysis, develop strategy advice for one of the players in the game or for a party interested in the outcome.

¹ The assignment is due on Lecture 10. If you are working on option 4, you may turn in your assignment late (up until the last class on March 11th).

*Sloan School of Management
Massachusetts Institute of Technology
Spring 2004, H1*

Qwest Bond Swap (Revised 2/13/04 for clarity)

DUE DATE

This assignment is due **at or before the beginning of class on February 19.**

INSTRUCTIONS

Turn in one solution per team.

All questions can be answered with just pencil-and-paper. Of course, you may choose to use a software program to help you (like Excel), but in this case please (i) provide documentation of what you did in your solution and (ii) email your program, spreadsheet, etc. to the TA.

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Sloan School of Management
Massachusetts Institute of Technology

15.040 - Game Theory for Managers
Professor David McAdams

Team Assignment #1: Qwest's Bond Swap

For this article, please see:

Norris, Floyd. "A Bond Swap Available Only To Big Players,"
The New York Times, 18 December 2002, C1.

Setup

In this homework we will examine this bond swap from a game theory point of view. In our (extremely simplified) model, each bondholder has one \$1000 Qwest bond. We ignore interest paid on the bonds and focus only on principal repayment. (We also ignore discounting.) We simplify matters further by

¹Floyd Norris, while certainly an excellent reporter of facts, is not a game theorist. His analysis and conclusions may or may not be correct.

assuming that Qwest has just one issue outstanding so that, prior to the tender offer, all bondholders have equal repayment priority. That is to say, if Qwest turns out to have X dollars in assets to pay back its bond obligations, then:

- Each bondholder gets back full face value of \$1000 as long as $X \geq \$1.5$ Billion. (And Qwest does not go bankrupt.)
- Each bondholder only gets back $\$1000 * \left(\frac{X}{1.5 \text{ Billion}}\right)$ if $X < \$1.5$ Billion. (And Qwest goes bankrupt.)

Qwest's tender offer allows any bondholder to trade her \$1000 bond for a \$500 bond that has repayment priority over those who do not tender. (\$545 from the article has been replaced by \$500 for numerical simplicity.) That is to say, suppose that after the tendering process the face-value of all outstanding newly-issued bonds is Y dollars and the face-value of all outstanding old bonds is Z dollars. Since each new \$500 bond-holder had to turn in a \$1000 bond, we know that $2 * Y + Z = \$1.5$ Billion.

- If $Y < X$, then all newly-issued bonds are repaid full face-value of \$500 and each old bond is partially repaid $\$1000 * \frac{X-Y}{Z}$.
- If $Y > X$, then old bonds get nothing and newly-issued bonds are partially repaid $\$500 * \frac{X}{Y}$.

Known Assets

In this section, we assume that there is no uncertainty about X , the value of assets that Qwest will have available to pay off its bonds.

1. Suppose that $X = \$500$ Million. Show that tendering is a dominant strategy for each bond holder.
2. Suppose that $X = \$1$ Billion. Show that NOT tendering is a dominant strategy for each bond holder.

Uncertain Assets

Qwest was not bankrupt yet at the time of the NY Times article's writing. And even if bankruptcy was viewed as being highly likely, there was certainly substantial uncertainty surrounding the exact value of Qwest's assets in the event of bankruptcy and hence uncertainty over how much bondholders would be paid back.

In this section, we treat the value of Qwest's assets, X , as a random variable. To get at the important ideas of the analysis without getting bogged down in complications, we will assume that X has a very simple distribution:

$$\Pr(X = \$500 \text{ million}) = 1 - p, \Pr(X = \$1 \text{ billion}) = p$$

The important features of this setup: Qwest is certain to go bankrupt if it can not get bondholders to tender but, if all bondholders tender, Qwest will escape bankruptcy with probability p . (If all tender, the face value of its debt will be only \$750 million.) Note that in questions 1 and 2 you analyzed the special cases in which $p = 0$ and $p = 1$, respectively.

3. For which p is it a dominant strategy to tender? Hint: You will find that it is a dominant strategy to tender as long as $p \leq p^*$ for some $p^* < 1$.
4. Your answer to question 3 shows that, ironically, Qwest can only hope to avoid bankruptcy through this tender offer if its chances are sufficiently small of escaping bankruptcy after a successful tender offer! Briefly explain (preferably without math) why this makes sense.

Up to this point, we have taken the terms of the tender offer as given. Suppose now that you are CFO of Qwest and must decide the terms of this tender offer. The CEO urges you to make the most favorable tender offer possible that is certain to succeed. (You interpret this as meaning that tendering should be a dominant strategy for bondholders.)

5. To be concrete, suppose that $p = 1/2$. What tender offer do you make?

Angry Madness: Payoff Detail

Your overall payoff will be determined by averaging your payoff in each of the 9 cases when you and your have $M, U = 100, 200, \text{ or } 400$. Your expected payoff in each of these 9 cases is, in turn, calculated by checking every possible way that the game might end.

Bart's expected payoff when $M = \$200$ for Ann and $U = \$200$ for Bart

We add up all of the terms below, which correspond to all ways the game might end:

First round, Bart yields	$50\% * \$200$	= 100
First round, Ann yields	$0\% * \$300$	
First round, both yield	$0\% * \$200$	
Anger after First round	$50\% * 10\% * \$0$	

(Note: Chances we go on to second round = $P1 = 50\% * 90\%$.)

Second round, Bart yields	$P1 * 25\% * \$200$	= 22.5
Second round, Ann yields	$P1 * 0\% * \$300$	
Second round, both yield	$P1 * 0\% * \$200$	
Anger after Second round	$P1 * 75\% * 20\% * \$0$	

(Note: Chances we go on to third round = $P2 = 50\% * 90\% * 75\% * 80\%$.)

Third round, Bart yields	$P2 * 75\% * \$200$	= 40.5
Third round, Ann yields	$P2 * 0\% * \$300$	
Third round, both yield	$P2 * 0\% * \$200$	
Anger after Third round	$P2 * 25\% * 30\% * \$0$	

(Note: Chances we go to fourth round = $P3 = 50\% * 90\% * 75\% * 80\% * 25\% * 70\%$.)

Third round, Bart yields	$P2 * 100\% * \$200$	= 9.45
Third round, Ann yields	$P2 * 0\% * \$300$	
Third round, both yield	$P2 * 0\% * \$200$	
Anger after Third round	$P2 * 0\% * 40\% * \$0$	

Bart's Expected Payoff = 172.45

... Bart would have been better off yielding for sure in Round 1 (which gives guaranteed payoff 200)

Ann's expected payoff when $M = \$200$ for Ann and $U = \$200$ for Bart

We add up all of the terms below, which correspond to all ways the game might end:

First round, Bart yields	$50\% * \$300$	= 150
First round, Ann yields	$0\% * \$200$	
First round, both yield	$0\% * \$200$	
Anger after First round	$50\% * 10\% * \$0$	

(Note: Chances we go on to second round = $P1 = 50\% * 90\%$.)

Second round, Bart yields	$P1 * 25\% * \$300$	= 33.75
Second round, Ann yields	$P1 * 0\% * \$200$	
Second round, both yield	$P1 * 0\% * \$200$	
Anger after Second round	$P1 * 75\% * 20\% * \$0$	

(Note: Chances we go on to third round = $P2 = 50\% * 90\% * 75\% * 80\%$.)

Third round, Bart yields	$P2 * 75\% * \$300$	= 60.75
Third round, Ann yields	$P2 * 0\% * \$200$	
Third round, both yield	$P2 * 0\% * \$200$	
Anger after Third round	$P2 * 25\% * 30\% * \$0$	

(Note: Chances we go to fourth round = $P3 = 50\% * 90\% * 75\% * 80\% * 25\% * 70\%$.)

Third round, Bart yields	$P2 * 100\% * \$300$	= 14.175
Third round, Ann yields	$P2 * 0\% * \$200$	
Third round, both yield	$P2 * 0\% * \$200$	
Anger after Third round	$P2 * 0\% * 40\% * \$0$	

Ann's Expected Payoff = 258.675

... Ann is better off with her strategy than yielding for sure in Round 1 (which gives guaranteed payoff 200)

Bluffing Game

We will play the Bluffing Game next class. As preparation, please think about the game, answer the questions on the opposite side, **and turn in this sheet to the TA at the beginning of next lecture.**

Description of Bluffing Game

The game is a very simple sort of poker. Be prepared to play either role (A or B)

- Both players put \$100 (hypothetical money) into “the pot” before play begins
- Player A receives a card. 1/3 of time it’s “Good Card”, 2/3 of time it’s “Bad Card”. (Player B does not get a card.)
- After getting the card, Player A can either bet \$100 more (“Raise”) or Not
- If Not Raise, game ends and Player A wins the pot (for net winning +100) if Good Card and loses pot (for net –100) if Bad Card. This is a zero-sum game, so B loses whatever A wins and vice versa.
- If Raise, Player B either gives up (“Fold”) or puts in an extra \$100 also (“Call”)
 - If Fold, Player A wins the pot automatically (payoff +100)
 - If Call, Player A wins the pot if Good Card (payoff +200) and loses the pot if Bad Card (payoff -200)

How would you play?

Suppose that, before the game is played, your opponent is given the chance to tell you “how she thinks the game should be played”. (Suppose for our purposes that she tells the truth.)

1. “The way to play is to Raise given a Good Card and Not Raise given a Bad Card. This maximizes one’s winnings and minimizes one’s losses.”

How would you play, given that this is how she thinks one should play? (Both if you are Player A and if you are Player B.) Why?

2. “The way to play is always Raise. That gives you the opportunity to win big. If you have a Good Card, then you can win +200 if opponent calls and if you have a Bad Card you can still win if your opponent Folds, getting +100 instead of –100.”

How would you play, given that this is how she thinks one should play? (Both if you are Player A and if you are Player B.) Why?

Your Name: _____

Dynamic Pricing Game

We will play the Dynamic Pricing Game next class. As preparation, please think about the game, answer the questions on the opposite side, **and turn in this sheet to the TA at the beginning of next lecture.**

Description of Dynamic Price War:

There are 100 customers and two firms.

Loyalty Stage: Each firm decides whether to create a loyal customer case. At a cost of \$250, each firm can guarantee that 30 customers will definitely buy its product regardless of the prices.

Pricing Stage: Coin-flip determines which firm goes first. Both firms start with tentative price equal to \$50. To keep things simple, firms are only allowed to set price equal to \$50, \$40, \$30, \$20, or \$10. The Pricing Stage ends as soon as both firms have had a move and someone announces “No Change”.

- If prices stay \$50 for both firms, each firm splits the market, selling 50 each
- If one firm ends up with a lower price, then the low-priced firm gets all customers who are not loyal to the high-price firm (either 100 or 70) while the high-price firm only sells to its loyal customers (0 or 30)
- If both firms end up with the same price, less than \$50, then whoever was first to set that price gets all customers who aren't loyal to the other firm (100 or 70) while that firm only sells to its loyal customers (0 or 30)

Payoffs in the Game. Each firm wants to maximize its own profit = revenue minus its cost for loyalty (either \$0 or \$250) ... firms have no other costs.

Example of play. (This example is not intended to illustrate good or bad play.) In Loyalty Stage, firm A creates loyalty while firm B does not. In Pricing Stage, Firm B wins the coin flip. It stays at \$50. Firm A then lowers its price to \$30. Firm B then lowers its price to \$20. Firm A then stays at \$30 ending the game. Firm A's profit is $30 * \$30 - \$250 = \$650$. Firm B's profit is $70 * \$20 - \$0 = \$1400$.

How would you play?

What would you do in the following two situations? Please briefly explain your answers.

1. You have invested in Loyal customers, but the other firm has not. They win the coin flip and decide to undercut you with price \$40.
How would you respond? Why?

2. Neither you nor your opponent has invested in Loyal customers. You win the coin flip. **Do you stay at \$50 or undercut? If undercut, what price? Why?**

3. Which do you plan to do: invest in Loyal customers or not?

Your Name: _____

Angry Madness

Today we are playing the Angry Negotiation Game in class. As an *optional follow-up*, you may enter our Computerized Angry Negotiation Tournament. To do this, respond to Professor McAdams' email later today.

Description of Computerized Angry Negotiation Tournament

You will play a *slight variation* of the game played in class. **After Rounds 1-9, people get angry with probability 25%** (rather than 10% after Round 1, 20% after Round 2, etc..) After Round 10, however, we still assume that people get Angry for certain.

Furthermore, half of the **overall** population of remaining players will survive each round. **(You and your opponent might both survive or both perish.)**

Gold Medal: bonus = 15% of total course grade
Silver Medal: bonus = 10% of total course grade
Bronze Medal: bonus = 5% of total course grade

How do you want your computer agent to play?

There are at most ten rounds. For each possible payoff you may get from yielding ($M = 100, 200, \text{ or } 400$)¹ specify **how likely you will be to Yield in each round, if that round is reached.** (See Example on next pages.)

	When M = 100	When M = 200	When M = 400
Round 1			
Round 2			
Round 3			
Round 4			
Round 5			
Round 6			
Round 7			
Round 8			
Round 9			
Round 10			

¹ To keep things simple, I'll use refer to you as Management. (This is business school, after all.) Half of you will play as "Labor", however, and half as "Management", but you don't know which.

Unlike February Madness, luck will not at all be a factor in this tournament. In each match, I will compute each player's *exact* expected payoff to determine the survivor.

Example: Ann vs Bart

Note: If you will be yielding 100% of the time if ever round R is reached, there is no way that round R+1 will be reached. That's why those boxes are empty.

Ann's Strategy

	When M = \$100	When M = \$200	When M = \$400
Round 1	50%	0%	100%
Round 2	100%	0%	
Round 3		0%	
Round 4		0%	
Round 5		0%	
Round 6		100%	
Round 7			
Round 8			
Round 9			
Round 10			

Bart's Strategy

	When U = \$100	When U = \$200	When U = \$400
Round 1	0%	50%	0%
Round 2	0%	25%	0%
Round 3	0%	75%	0%
Round 4	100%	100%	0%
Round 5			0%
Round 6			0%
Round 7			0%
Round 8			0%
Round 9			0%
Round 10			0%

What Will Happen If $M = U = \$200$

In Round 1, Ann never yields and Bart yields 50% of the time

- 50% of the time, Ann gets \$300 and Bart gets \$200
- $50\% * 10\% = 5\%$ of the time, Ann and Bart get angry (and \$0 payoff each)
- So, Round 2 is reached only 45% of the time

If Round 2 is reached, Ann never yields and Bart yields 25% of the time

- 25% of the time, Ann gets \$300 and Bart gets \$200
- $75\% * 20\% = 15\%$ of the time, Ann and Bart get angry (and \$0 payoff each)
- So, Round 3 is reached from Round 2 only 60% of the time. All together, Round 3 is reached only $45\% * 60\% = 27\%$ of the time

If Round 3 is reached, Ann never yields and Bart yields 75% of the time

- 75% of the time, Ann gets \$300 and Bart gets \$200
- $25\% * 30\% = 7.5\%$ of the time, Ann and Bart get angry (and \$0 payoff each)
- So, Round 4 is reached from Round 3 only 17.5% of the time. All together, Round 4 is reached only $27\% * 17.5\% = 4.7\%$ of the time

If Round 4 is reached, Ann never yields and Bart yields 100% of the time

- Ann gets \$300 and Bart gets \$200
- Round 5 is never reached

Given their payoff in every possible event, we can calculate each player's expected payoff when $M = U = \$200$ and, similarly, in all of the other cases ($M = U = \$100$ and $M = \$100, U = \200 , etc..) The average payoff across all nine cases is what determines who survives to the next round.

For more details, feel free to examine the materials on Sloanspace:

1. Word document that breaks out in more detail the calculation for the highlighted case $M = U = \$200$ above.
2. Excel spreadsheet that will be used to compute the payoffs. (Feel free to experiment – input various strategies and see what overall payoffs result.)

15.040, Spring 2004
Summary of Online Game 1
Courtesy of Mike Shor, Vanderbilt University

This game is based on the Miss Rheingold 1957 beauty contest. You have the chance to vote for one woman from the field of contestants who are all vying for the title of Miss Rheingold. You are eligible for the \$50,000 jackpot if you vote for the contestant who wins. Who will you choose?

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15.040, Spring 2004
Summary of Online Game 3
Courtesy of Mike Shor, Vanderbilt University

Game 3: An extensive form game demonstrating subgame perfection

This timed game consists of two decisions that you have to make:

1. You are considering entering an industry currently dominated by a monopolist (a randomly selected classmate). You are the only possible entrant into this market. You must decide whether to enter or stay out, given the possible responses to entry available to the monopolist. What action do you select, enter or stay out?
2. Now, you're the monopolist and an opponent is deciding to enter your industry or not. Suppose that the other firm chose to enter your industry. Do you accommodate the entry, or do you fight?

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15.040, Spring 2004
Summary of Online Game 4
Courtesy of Mike Shor, Vanderbilt University

Game 4: A game without pure strategy equilibria demonstrating mixed strategies

In this timed game, you have been hired as a consultant to put together a strategy for a new product release. As a consultant, you get paid well before the company knows how hard you worked on the project. The company has a "random employee monitoring program", through which it randomly selects some consultants to check on whether or not they are actually working. If they are caught shirking, or not working, they don't get paid. However, if you choose to work, you have to give up a side job, and hence working has a "cost." Do you choose to work or to shirk?

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15.040, Spring 2004
Summary of Online Game 6
Courtesy of Mike Shor, Vanderbilt University

Game 6: Demonstrates option value and commitment under uncertainty

In this timed game, you are a company at the forefront of a new digital television technology. You and one competitor (another student in class) have an innovator's advantage as you are the main developers of the new technology. Your advantage makes it possible for you and your one leading competitor to build production plants right away, committing to market entry. A number of other firms will be ready to enter soon if you do not take advantage of your lead. Your problem is that you cannot guess the likelihood of the commercial appeal and popular adoption of the technology. Do you enter now or wait to see if the technology gains popular acceptance?

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15.040, Spring 2004
Summary of Online Game 7
Courtesy of Mike Shor, Vanderbilt University

Game 7: A mechanism design (principal agent) problem

In this timed game, you are the owner of a large pharmaceutical company looking to subcontract out a number of research projects, each with uncertain outcome, to outside R&D facilities. Each research facility, once hired, can choose either to put in routine effort (assign a normal staff to the project) or high effort (assigning its best researchers). Its choice will affect how likely the project is to succeed and earn a profit. You cannot monitor a facility's effort directly; all that you know is whether or not its project succeeds. You must offer all research facilities the same offer consisting of a base salary and bonus if the project is a success. What salary and bonus do you wish to offer?

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15.040, Spring 2004
Summary of Online Game 8
Courtesy of Mike Shor, Vanderbilt University

Game 8: An auction demonstrating the winner's curse

This timed game consists of two decisions that you have to make:

You are the Chief Financial Officer for your firm, which is considering making a takeover offer for a small, privately-held Internet startup. The "true" value of the shares of the startup are known with certainty only to its management. You only know that the block of shares is worth an amount within a given range. You are required to recommend to your firm the amount that should be bid for the startup. You are the only bidder and this is your only chance - if your offer is rejected, there is no room for renegotiation. How much do you bid per share? The second decision you have to make has the same setup as the first, with a different range of values.

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Urn Game

We will play the Urn Game next class. As preparation, please think about the game, answer the two questions at the bottom of this page, **and turn in this sheet to the TA at the beginning of next lecture.**

Description of Urn Game:

- “Urn W” has two white balls and one yellow ball
- “Urn Y” has one white ball and two yellow balls.
- The TA flips a coin and picks up Urn W if Heads and Urn Y if Tails.
- Students are invited – one at a time – to pick a ball from the chosen urn and then guess whether it’s Urn W or Urn Y. The class gets to learn each student’s guess BUT NOT what type of ball the student saw. *The chosen ball is then put back into the urn.*
- Payoffs: You “WIN” if you guess correctly and “LOSE” if you guess incorrectly.

How would you play?

What would you do in the following two situations? Please briefly explain your answers.

1. You are the third person to pick a ball. The first person and the second person both guessed “Urn W”. You draw a yellow ball.
Would you guess “Urn W” or “Urn Y”?
2. You are the tenth person to pick a ball. The first seven people each guessed “Urn W” while the next two people each guessed “Urn Y”. You draw a yellow ball.
Would you guess “Urn W” or “Urn Y”?

Your Name: _____

Angry Negotiation Game

We will play the Angry Negotiation Game next class. As preparation, please think about the game, answer the questions on the opposite side, **and turn in this sheet to the TA at the beginning of next lecture.**

Description of Angry Negotiation Game:

There are two players, Union and Management, in an all-or-nothing negotiation over a specific issue. (No compromise is possible.) The negotiation continues until some side “Yields” or they get “Angry”, after which no agreement is possible. More specifically:

Payoffs in the Game: Each player prefers most to reach agreement by having the other person yield, but would rather Yield itself than fail to reach agreement. In particular,

	Management Payoff	Union Payoff
They get Angry	0	0
Union Yields, Mngt Doesn't	$M+100$	U
Mngt Yields, Union Doesn't	M	$U+100$
Both Yield	M	U

M and U are each initially unknown variables, with each equal to 100, 200, or 400 with probability 1/3. Before the game starts, Management learns M and Union learns U (but Management does not learn U and Union does not learn M).

Multi-Round Negotiation: Each round, the two parties simultaneously decide whether to Yield. If either or both Yield, the game ends. Otherwise there is a chance that they will get Angry. After the first round, they get angry with probability 10%, after the second round they get angry with probability 20%, after the third 30%, and so on. If they do not get Angry after a given round, on the other hand, the game continues to the next round with each again deciding whether to Yield.

Example of play. (This example is not intended to illustrate good or bad play.) Union learns $U = 400$ and Management learns $M = 200$. In Round 1, both choose Don't Yield. The 10% chance of Angry does not happen so we continue. In Round 2, both choose Don't Yield. The 20% chance of Angry does not happen so we continue. Finally, in Round 3, Union Yields but Management does not. Game ends with Union getting payoff 400 and Management getting payoff 300 ($200+100$).

How would you play?

Suppose your opponent says one of the following quotes to you, just before playing.

First, is your opponent correct? Second, what would you say back / do during the game?

1. “Even if you have the highest value for avoiding failure (400), the prospect of winning an extra 100 is worth the 10% risk of losing 400 in Round 1. So, no one should ever Yield in Round 1.”
2. “The way to play this game is to tell the other person that you are never going to Yield. That forces them to Yield (and Yield immediately). That makes sense, right? Well, just so you know, I’m never going to Yield.”
3. Suppose your opponent is Professor McAdams and you are able to make one of the above statements before playing, but he is not allowed to respond. Which would you say (you need not believe what you say)?

Your Name: _____

Sloan School of Management
Massachusetts Institute of Technology
Spring 2004, H1
Professor David McAdams



15.040 Practice Final Exam

The open-book take-home final exam will be similar to this practice test with three problems:

1. Describe a game having certain features and analyze it.
2. Problem that requires you to analyze the equilibria in a game.
3. Problem that requires connecting concepts from the course in a given scenario.

1. Describe a two-player game in which (i) each player regards the strategies as strategic complements, (ii) player A prefers that player B adopts a “more aggressive” strategy and (iii) player B prefers that player A adopts a “less aggressive” strategy.

- a. What is each player's reaction curve in your game?
- b. Compute all Nash equilibria in your game given simultaneous moves.
- c. Compute all subgame-perfect equilibria in your game when player A moves first as well as when B moves first.
- d. Which would each player prefer, to move first or to move last?

2. Male elk are known for their battles at the start of mating season. Whenever two males (A and B) meet, they always spar a bit with their antlers but ultimately each makes a choice of whether to back off or to fight to the death. If each backs off, each has 90% probability of finding a mate. If A backs off but B does not, then A has 80% chance of mating while B has 100% chance of mating. If neither backs off, then they fight to the death and only the survivor mates (50% chance of mating).

- a. Describe all Nash equilibria of this game. (Be sure to include the mixed strategy equilibrium.)
- b. Which of these equilibria are evolutionarily stable?

[Practice final exam continues on next page.]

¹ If you can not think of a game satisfying (i)-(iii), then for partial credit answer parts (a)-(d) for some other game, preferably one that we have not studied in the class.

3. In 2020, Sony enjoys a monopoly in the market for video game consoles. Every five years, however, Sony is vulnerable to entry when it introduces a new version of its Playstation console, but Sony has been able to deter entry through its cost advantage due to learning. In particular, by producing much more than the monopoly-optimal quantity in each product cycle (giving up short-run profits), Sony keeps its costs low enough to deter entry in the next cycle.

Also in 2020, your firm has secretly developed a revolutionary new computer chip (based on quantum-tunnel transistors) that will allow you to profitably enter the video game console market in 2025 -- regardless of how much Sony produces in the 2020 product cycle.

Is it better for you to reveal your plans to enter the video game console market now or to keep them secret until 2025? Be sure to indicate whether your answer depends on whether future competition in this market is in strategic substitutes or strategic complements, or other factors.

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Solution to Practice Exam, 15.040 Spring 2004

Problem 1

Problem 1 on the final exam, will be very similar to this, though the conditions will be altered.

Thoughts on how to approach this problem:

Problem 1 has two parts. First, you must come up with a game having certain features. Second, you must analyze the equilibria of this game. The second part is more straightforward, since you can use the blueprint provided by the analysis of examples in the textbook, in the Lecture Note on Strategic Substitutes and Complements, and in the lecture slides.

How then to approach the first part? The question asks us to think of a game in which (a) both players regard the strategies as strategic complements and (b) player A wants player B to be more aggressive whereas (c) player B wants A to be less aggressive. One natural way to approach this problem is to think first of games we have seen in which both players regard the strategies as strategic complements, and then to think about how a simple variation on one of those games might satisfy the other conditions. In this case, the Cooperation Game from the Lecture Note is an example in which both players view the strategies as strategic complements, though both players prefer that the other be more aggressive (i.e. do more work).

Is it possible to plausibly modify the Cooperation Game so that one of the players prefers the other to be less aggressive, while keeping the strategic complementarity? Strategic complementarity in this game means that each worker wants to work hard if the other works hard but not otherwise. In the original Cooperation Game, the reason for this is that each worker is only willing to put in effort if that will lead to the job getting done. Thus, you prefer that the other player works harder since then more of the task gets

done. But another natural reason to work only if the other player works is to avoid looking bad to the boss. In that case, you will prefer that the other player works less since then you will look relatively better. Let's try to create a very simple game based on this idea. (Having a simple game, obviously, simplifies the analysis in the second part.)

Solution

The Game: "Working with a Slacker". Two workers. Each has a simple choice, Work or Not. If both Work the job gets done well, but if one or both of them do not work then the job gets done poorly. Worker **A** (Alice) only wants to work if doing so will lead to a well-done job. In particular, having a well-done job is "worth 10" and working "costs 5". Worker **B** (Bob) only wants to avoid looking bad. In particular, while working "costs 5", being the only one not to work "costs 10". (Note that, holding her own action fixed, Alice prefers for Bob to work harder whereas, holding his action fixed, Bob prefers that Alice work less.¹) Putting this together, we get the payoff matrix

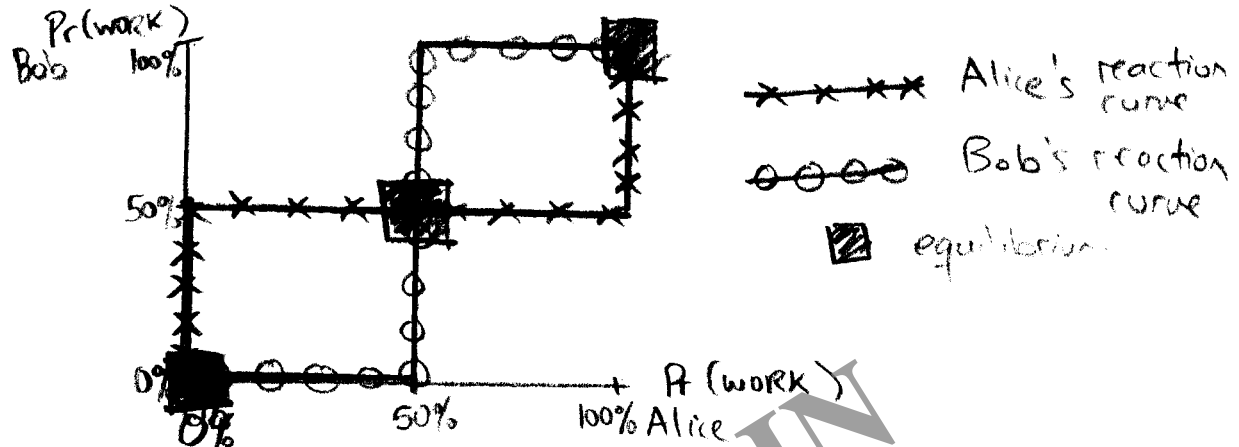
		Bob	
		WORK	NOT
Alice	WORK	5, -5	-5, -10
	NOT	0, -5	0, 0

(a) Reaction curves. For Alice: $R_A(WORK) = WORK$ and $R_A(NOT) = NOT$. For Bob: $R_B(WORK) = WORK$ and $R_B(NOT) = NOT$.

(b) Nash equilibria given simultaneous moves. Clearly, (WORK, WORK) and (NOT, NOT) are Nash equilibria. What about mixed strategies? By working, Alice is "risking 5 (payoff -5 rather than 0) to gain 5 (payoff 5 rather than 0)" so for her to be indifferent Bob must work 50% of the time. Similarly, by working Bob is "risking 5 (payoff -5 rather than 0) to gain 5

¹In the Practice Final and in the Lecture Note on Strategic Substitutes and Complements, I routinely refer to games in which "player *A* wants player *B* to be more aggressive", etc. This means that, holding player *A*'s action fixed, player *A* is never worse off (and sometimes better off) when player *B* chooses a higher ("more aggressive") strategy.

(payoff -5 rather than -10)" so he is indifferent when Alice works 50%. So, (WORK 50%, WORK 50%) is the (only) mixed-strategy equilibrium. We can also see the three equilibria by drawing the players' reaction curves.



(c) Subgame-perfect equilibria given sequential moves. *When Alice goes first:* As the follower, Bob's strategy is determined by his reaction curve: WORK if Alice WORKs and NOT if Alice NOTs. Given this, Alice chooses to WORK since she prefers (WORK, WORK) over (NOT, NOT). *When Bob goes first:* As the follower, Alice's strategy is determined by her reaction curve: WORK if Bob WORKs and NOT if Bob NOTs. Given this, Bob chooses NOT since he prefers (NOT, NOT) over (WORK, WORK).

Note: to be fully correct, you must specify a player's action at every one of his information sets. For example, when Alice is leader Bob has two information sets: (1) Alice WORKs and (2) Alice NOTs. In equilibrium, we don't observe what Bob was planning to do if Alice didn't work, but his plan of action in that unrealized event is an important part of the equilibrium.

(d) Preferences over moving first vs. last. *For Alice:* When Alice moves first, we have the outcome (WORK, WORK), whereas when Alice moves last we have the outcome (NOT, NOT). Thus, moving first is better than moving last. *For Bob:* When Bob moves first, we have the outcome (NOT, NOT), whereas when Bob moves last we have the outcome (WORK, WORK). Thus, moving first is better than moving last.

The results of the previous paragraph are to be expected. Take Alice: since Bob views the strategies as strategic complements and Alice wants Bob to work more, we know that she will tend to work more as the leader than when moving simultaneously. This makes Bob worse off and explains why he

prefers not to be the follower. Similarly, since Alice views the strategies as strategic complements and Bob wants Alice to work less, we know that Bob will tend to work less as the leader than when moving simultaneously. This makes Alice worse off and explains why she prefers not to be the follower.

Problem 2

Problem 2 on the final exam will be similar to this, but may involve a different type of game.

Solution

(a) This is an example of the Chicken Game; see Slides for Lecture #5 and the reading in the textbook. We know that there are two pure strategy equilibria, (Back Off, Fight) and (Fight, Back Off), as well as a mixed strategy equilibrium. What are the fighting probabilities in this mixed strategy equilibrium? Each Elk is "risking 30% (80% - 50% if the other fights) to gain 10% (100% - 90% if the other backs off)". So to be indifferent between fighting and backing off, the probability of fighting must be 25%. (Because $25\%/75\% = 1/3$, the opposite ratio of the risk and reward from fighting.) Thus, the mixed strategy equilibrium is (25% Fight, 25% Fight).

(b) Since this is the Chicken Game, we know from Lecture that the answer to this question depends on whether male Elk evolve as one population or as two populations. What does this mean? Remember that evolution works by rewarding players who do relatively well compared to an average population, but what population is that? If it is the population of all players (both those who act as player A and those who act as player B), then there is one population. If it is just the population of players in the same role (only the player A's if you are player A) then there are two populations. In this case, male elk are successful evolutionarily if they have more offspring than other male elk – which elk "acts as player A" doesn't matter. So, there is evolution of just one population. In this case, theory tells us that *only the mixed strategy equilibrium is evolutionarily stable*.

Problem 3

Problem 3 on the final exam will be similar to this, in that it will be based on a real (or "real") business strategy problem for which you will be asked to

provide strategy advice.

Solution

There are several issues that might be raised in the solution to this problem. I will focus on surprise and how strategic substitutes and complements relate. (See Lecture Note on Strategic Substitutes and Complements as well as the slides for Lecture 6.)

In our firm's future competition with Sony, we want Sony to be less aggressive. (In the terminology introduced in Lecture 6, our competition with Sony is a "competitive game" rather than a "reliance game".) Suppose first that this game is one in **strategic substitutes**. From page 14 of the Lecture Note, then, we prefer moving simultaneously to surprise. Since we also prefer moving first to moving simultaneously (page 11), it's clear that we should reveal ourselves and - if possible - commit to our strategy as soon as possible.

What if the game is one in **strategic complements**? Here we know that we prefer surprise to moving simultaneously. The only question, then, is whether surprise is better than leadership as well. This question is not addressed in the Lecture Note, so to make progress we need to think about why leadership is better than moving simultaneously. As the leader, we will commit to a strategy that is less than our Nash equilibrium strategy, thereby inducing Sony to also play a strategy that is less than his Nash equilibrium strategy. But wait! In a world with surprise, Sony acts as if we have committed to the least aggressive strategy of all! There is no way, through leadership, for us to induce Sony to play a less aggressive strategy than what they are already planning to do. On the other hand, surprise gives us the flexibility to play a best response to Sony's strategy (rather than committing to a strategy that is not a best response). Putting this together, surprise is definitely better than either moving simultaneously or moving first when future competition is in strategic complements.

Further discussion of strategic substitutes / complements

This is all well and good, but it is very abstract. What will future competition be in, strategic substitutes or complements? First, some generic examples of types of competition that tend to fit into either category:

- *Compete on capacity/quantity:* Tends to be strategic substitutes. Example: Georgia Pacific and other lumber companies decide how much capacity to build to make processed lumber products, and then produce up to capacity, selling what they make at the market price.
- *Compete on price:* Tends to be strategic complements. Example: FedEx and other package delivery services set their own prices and then delivery packages for all customers who choose them.
- *Compete on advertising:* Tends to be strategic complements. Example: Pfizer (Viagra) and other makers of drugs for erectile dysfunction attract business through extensive marketing campaigns aimed at doctors as well as campaigns aimed at potential patients.
- *Compete on research:* Can go either way. In a winner-take-all patent race, for instance, the nature of competition depends on how close the race is. When the race is close, each player tends to view the strategies as strategic complements ("I can't let them take the lead"). When one player is way ahead, however, the player who is behind tends to view them as strategic substitutes ("why put in effort if other is putting in huge effort") while the player who is ahead tends to view them as strategic complements ("I must maintain my lead"). For examples of competition on research, think not only of the pharmaceutical market but the film industry, the toy market, etc...

We have a student from Sony in our class, who can answer this question better than I can, but here is my stab at an analysis. In today's market, Sony, Nintendo, and Microsoft's short-run competition seems to be mainly in prices and in advertising, which would correspond to strategic complements. The problem, however, asks us to think about announcing entry that is five years away = what's important for this is not the nature of short-run but of long-run competition. Here, I would say that they compete primarily on research and that this research is always a "close race", i.e. probably best viewed as being in strategic complements. Consequently, the firm should not announce its intentions at this stage. Not surprisingly, this advice has a very natural intuition: "if we announce our future entry, Sony will put its research engine into overdrive and reduce our advantage by developing more new games, etc., even if they can't match our quantum-tunnel production technology".